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Video-Cued Parental Dialogs: A Promising Venue for Exploring Early Childhood Mathematics *

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Abstract

Research literature to date provides limited insights about the mathematics learning of young children developed within the context of family, and families' perspectives on young children's mathematics. As we push to develop better mathematics learning opportunities for young children, parents seem to be left out of the picture. This study, eliciting parental voices through multi-vocal video-cued interviews, showed parental insight about the processes of their four and five year old children engaging with mathematics out of school, with particular interest in mathematics content ranging from number sense to data analysis, and parents' pedagogical ways of and concerns about contextual aspects of their children's mathematics. Findings from this research indicate that teachers' video-cued dialogs with parents could have potential to inform practice, at first in acknowledging parents-as-intellectual resources, and then, in creating spaces where teachers and parents might come together and engage in learning from each other about ways to create more diverse mathematics learning opportunities for young children in and out of school. These dialogues could help teachers to make curricular connections with parents, as they inquire together about how children engage in and transform mathematical practices as they move back and forth between home and school.

Keywords

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Introduction

A growing body of research has demonstrated that children develop significant mathematical understandings in their early years (Baroody, 1987; Clements, Sarama, & DiBiase, 2004; Charlesworth, 2005; Gingsburg, Klein, Starkey, 1998; Strauss & Curtis, 1981) and that these early experiences greatly affect children's later mathematical learning (Clements & Sarama, 2000; Duncan et al., 2007; van Hiele-Geldof, 1959/1984). These studies have shaped our thinking about young children's potential and the manner in which they develop mathematical thinking in their early years. Given this work, the importance of high quality mathematics education during the early childhood period has gained greater prominence.

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In the last decades, interdisciplinary research has revealed that beyond the earlier conceived notion of developmental stages (Kamii & DeClark, 1985), the way in which young children experience mathematics is a complex and multifaceted process that is situated within the dimensions of the socio-cultural contexts of learning. In particular, in the last decades there has been a resurgence of interest in these socio-cultural aspects of children's mathematics learning processes and an expansion of the ways in which cultural context, equity issues, race, social class, and politics are theorized and studied (Allexsaht-Snider & Hart, 2001; Apple, 1992; Parks, 2010; Tudge & Doucet, 2004). The literature that draws on these broad issues has brought an extended understanding to children's mathematics to the dynamics of our understandings of the contexts in which the mathematics learning process takes place within the interactions among children, teachers, parents, and the community.

In the search for new directions and re-conceptualizations of early childhood mathematics education, calls are made with the hopes of discovering approaches to better teaching of mathematics for young children (National Association for the Education of Young Children & National Council of Teachers of Mathematics, 2002). These calls for advancements in early childhood mathematics education are centered around the idea of supporting teachers of young children. These calls have foregrounded the idea that meaningful mathematical opportunities for young children can be created by equipping teachers with the skills to recognize everyday mathematically rich moments of young children and turn them into learning moments, as well as by developing purposeful instruction of mathematics in early childhood classrooms (Hachey 2013; Stipek, 2013). Although these are promising paths to follow, this perspective confines our view of rich mathematical learning opportunities for children to their social interactions with their teachers in the context of schooling. In other words, the family context for mathematically rich learning opportunities for young children seems to be missing from the picture in the effort to envision better mathematics learning opportunities for young children.

The research we present in this paper considers the possibility of ways in which parents of young children might contribute to our understanding of children's mathematical potentials and ways to support their mathematics learning. In this study, we were interested in eliciting parents' visions of their children's mathematics in order to re-envision what might constitute meaningful mathematical practices and learning environments for young children. To elicit parental dialogs on early childhood mathematics, in this research I (first author) conducted multivocal video-cued interviews (described later) with families. In short, I recorded children's mathematical engagements in a Prekindergarten (Pre-K) classroom in Spring 2013 and then, shared these videos with their families to generate dialogs about children's mathematics both at home and in school.

Literature Review

Young children's mathematical practices both in and out-of school contexts is highlighted in the educational literature; however, much of this research has underestimated the potential role of parents in mathematics education. Cross, Woods, and Schweingruber (2009) points to the gap in the literature in terms of empirical research on parental beliefs about young children's mathematics at the same time recognizing that parents play important roles in supporting children's mathematics, in particular numeracy-related learnings at home, developing attitudes towards mathematics, and preparing an appropriate learning environment through providing rich materials in out-of-school contexts. Similar calls for parental engagement in children's mathematical learning have echoed in the educational literature, reflecting the recognition of the importance of mathematics in children's lives and the instrumental role of mathematics in society (Cai, 2003; Goos & Jolly, 2004; Dávila-Coates & Franco, 1999; Saxe, Guberman, Gearhart, 1987; Sheldon & Epstein, 2005). These calls are foregrounded in the recent research on family engagement in mathematics, which presents positive outcomes for both children and diverse families, as well as for teachers (Allexsaht-Snider, 2006; Baker, Street, & Tomlin, 2006; Blevins-Knabe & Musun-Miller, 1996; Civil, Planas, & Quintos, 2005; Martin, 2006).

In a review of the limited but growing body of research on family engagement in mathematics during the early childhood period, a number of studies claim that the dominant discourse about young children's language and literacy development inhibits families' engagement in their children's mathematical development and limits the valuing of mathematics in the early years (Barbarin, Early, Clifford, Bryant, Frome, Burchial, Howes, & Pianta, 2008; Blevis-Knabe & Musun-Miller , 1996; Cannon & Ginsburg, 2008; Plewis, Mooney, and Creeser, 1990). These studies either present information coming from parents about their beliefs related to mathematics or present parents' reported observations about children's mathematics in out-of-school contexts. However, it seems that the methodologies employed in research with families might be restricting what we can learn from parents in terms of their children's mathematics and the way families engage in their children's mathematical learning. When parents are asked directly about children's learning, language and literacy practices of young children may be much more visible to detect compared to mathematics (Tudge & Doucet, 2004).

Focusing on how parents of young children can have influential roles on children's mathematics, a growing body of research provides valuable insights. Not surprisingly, the more families provide cognitive support and home resources explicitly featuring mathematics (such as puzzles, shapes, and blocks) at home, and the more they actively engage in recognized efforts to explicitly teach mathematics at home, the more their children's early mathematical ability is increased (Powell, Son, File, & San Juan, 2010). Moreover, families' positive attitudes ttoward mathematics also seemed to have positive effects on children's mathematics (Skwachuk, 2009), Similarly, Blevins-Knabe and Musun-Miller (1996) found that families' reported frequency and variety of mathematics-related activities at home have connections with their children's mathematical ability measured by a standardized test, and also predicts their future mathematical abilities (Vukovic, Roberts, & Green-Wright, 2013). A very recent extensive literature review from the past decade also shows that engaging families in early mathematics has a positive connection with children's mathematical abilities in preschool, kindergarten and the early elementary grades (Maier, 2014). These studies are enlightening in terms of showing the favorable effect of parental influence on children's mathematical understandings, particularly in experimental research conditions or intervention-based research. Yet we still need more research to understand the nature of parental support in mathematics in children's everyday lives and how to build bridges between parents and teachers to think about new ways of enhancing young children's mathematics both in and out-of-school contexts.

Some of our important insights about socio-cultural influences in mathematics learning are rooted in research with diverse families and children that examine the wide range of influences on children's mathematics including social class, ethnicity, race, and gender. This research has shown that regardless of their backgrounds, parents are involved in mathematical engagements with their children (Saxe, Guberman, & Gerhalt, 1987; Ginsburg & Russell, 1981; Starkey & Klein, 2000). However, there has been an increasing tendency (or assumption) in the current literature in terms of associating young children's *lack of readiness* in mathematics with their families' social class and racial backgrounds, although the research results on this issue are not conclusive (Parks, 2014). Another body of research with very young children and their families, employing ethnographic perspectives on the lives of young children, has critically questioned the gap between schools and home in terms of the expectation of what constitutes mathematics, and what are the legitimate ways of teachings mathematics to young children. These promising studies (Acar, 2010; Anderson & Gold, 2006; Baker, Street & Tomlin, 2006; Parks & Bridges-Rhoads, 2009) demonstrate that more understanding of the value of familial resources is needed when shaping our mathematical expectations for young children. The researchers also argue that mathematical practices of families can be overlooked because of the methodologies employed to conceptualize and understand the families' efforts to develop and enhance children's mathematics.

To summarize, research literature to date provides some initial insights into the mathematical abilities of young children developed within the context of family, and families' perspectives on young children's mathematics. Drawing on this previous work, the starting point in this study was to position families as sources of knowledge and to generate dialogs with them about the idea of what is valuable is terms of mathematics in children's daily lives. The present study, employing multi-vocal video cued interviews, in which videos of children taking part in diverse mathematical engagements in their preschool settings, both explicit and not-so-obvious, were shared with their parents to be able to elicit rich conversations on mathematics. The video-cued interviews were used as an alternative to directly questioning parents about their children's mathematical engagements, since that approach had been identified as problematic in earlier research. Grounded in Vygotskyian (1978) theoretical perspectives, and drawing on the notion of parents as intellectual resources (Civil & Andrade, 2003), the research we present in this article situates parental voices, elicited through multi-vocal video-cued interviews (Tobin, Wu, & Davidson, 1989), as sources of knowledge that have power to shed light on the diversity of young children's mathematical engagements in both out-of-school and in-school contexts. Particular research questions, which guided this study, are: How can video-cued multivocal parental interviews assist us to engage in dialogs with parents about their children's mathematical experiences? In the context of video-cued multivocal interviews, what do parents of young children tell us about diverse in- and out-of-school mathematical experiences of children?

Theoretical Framework

Of particular importance in thinking about young children's mathematics are the social interactions in which they engage, starting from the very early months of their lives. Vygotsky (1978) wrote that, "Human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them" (p.34). From a Vygotskyian perspective (1978), children's cognitive constructions, in other words, not only their actual but particularly their *potential* developmental learnings, come to life through their social interactions that take place in a context in which socio-cultural values, historical factors, and language play fundamental roles. Grounded in this Vygotskyian perspective, our research privileges the social interactions composed of interplays between mathematical dialogs and activities among parents and young children as valuable resources for their learning, and focuses on the nature of assistance reported by families during these social interactions.

Vygotsky (1987) proposes that children's concept formation manifests itself on two different levels: everyday (or spontaneous) concepts and scientific (schooled) concepts. Everyday concepts are those a child acquires by himself/herself through interacting with the world and characterize the child's actual development, whereas scientific concepts are those a child acquires only through the social interactions with more capable adults or peers and refer to a child's potential development. Vygotsky (1978) conceptualizes the dynamic state between these actual and potential developmental learnings of children as the zone of proximal development, and emphasizes the importance of providing rich and stimulating experiences to children functioning in this zone.

The question of how and what kind of mathematical concepts children develop in these zones may generate different answers considering the nature of social interactions in the communities of practice (Lave & Wenger, 1991) in which the mathematics take place. Considering school and home as different communities of practice, for example, suggests that as these contexts change, the mathematics that is being produced in these communities of practice also changes because the practices and dialogs around the mathematics differ as different communities of practice possess differing daily cultural practices (Lave & Wenger, 1991). González, Andrade, Civil, and Moll (2001), for example, propose that "hegemony of a particular type of mathematics" (p.124) in school contexts may prevent educators from recognizing the rich mathematical experiences of children in out-of-school contexts.

Considering the Vygotskyian conceptualization of the everyday and scientific concept formation of children, González and colleagues (2001) remind us that children's everyday mathematical concept development should not be ascribed solely to their household practices, with their scientific concept development ascribed solely to their in-school mathematical experiences. Put succinctly, as children experience different mathematical practices in- or out-of-school, their mathematical concept formation, either everyday or scientific, continues to develop regardless of the context. In line with this integrated and comprehensive perspective, rather than drawing a distinction between mathematical practices in these two contexts, our research aims to understand the multidimensionality of children's mathematical experiences, residing in their social interactions in- and out-of-school as a whole, and situates parental voices as sources of knowledge that have power to shed light onto the diversity of young children's mathematical engagements.

Civil and Andrade's (2003) family engagement framework, which draws on socio-cultural theoretical perspective of Vygotsky (1978), provided strong foundations to our study in theorizing the potentials of *"parents as intellectual resources"* about young children's mathematics. Civil & Andrade (2003) problematizes the traditional family engagement practices in schools and propose that parents hold valuable potential as intellectual resources for their children's mathematical learnings. Therefore, family engagement frameworks in content areas such as mathematics should be conceptualized beyond restricted notions such as attending school-oriented math clubs or family math nights. In their longitudinal research titled MAPPS (Math and Parent Partnership in the Southwest), Civil and colleagues (2006) embraced a funds-of-knowledge perspective (Moll, Amanti, & González, 2005), and created a family engagement framework that centered around the notion of parents as intellectual resources by recognizing the various effective roles of parents, such as parents as (a) parents, (b) learners, (c) teachers and (d) leaders. The findings in our study provide perspectives in line with this framework, which posits that families hold potentials to teach mathematics to children in diverse ways. Parents show eagerness to learn about the ways to support their children's mathematics; they are eager to take leadership roles in their children's mathematical learning; and as parents they bring their own close observations of the mathematical learning process of their children.

Methodology

In this study, our research design was guided by "video-cued multivocal ethnography," a method developed by Tobin and his colleagues in the study *Preschool in Three Cultures: Japan, China and the United States* (1989) and also in the follow-up study *Preschool in Three Cultures Revisited: China, Japan and the United States* (Tobin, Hsueh, & Karasawa, 2009). In the original studies, researchers videotaped a typical day in a preschool in each culture, and then used these videos as cues to generate dialogs with early childhood educators across the countries with the aim of eliciting their own pedagogical practices embedded in their cultural beliefs. Drawing on this methodology, I recorded diverse mathematically rich engagements of four- and five-year-old children in a Pre-K classroom, and then, I shared these videos with families during the multivocal video-cued interviews. These interviews generated parental dialogs about children's mathematical engagements both occurring in the preschool and also out-of-school contexts.

Research Context

I recorded the videos of the video-cued multivocal interviews in a university-affiliated childcare center because of the ethnically diverse profile of children in the classrooms and the convenience for video-based research at the site, which is located in the southeastern United States. This childcare center is open to infants, toddlers, and preschoolers; I conducted the study with families of the Pre-K children between the ages of 4 and 5. Focusing on the classroom where I recorded the videos, the class had three teachers, one of whom was the lead teacher and the other two were assistant teachers. The mathematics center of the classroom was composed of a medium-sized rug and a wooden shelf holding mathematical materials such as items for sorting, board games, clock toys, measurement tapes, hourglasses, calendars, and small cubes. Although there was a specified center for mathematics, evidence of mathematics engagements could be observed everywhere in the

classroom through the design of the environment, and in both teacher-directed activities and children's self-directed engagements. Counting was largely embedded in teacher-directed math activities, and in daily classroom routines during gym, snack, and calendar time, such as, for example, counting the sunny, windy, or snowy days in the calendar where they kept record of this information. During the research process, the main teacher-directed mathematical activities were grounded in number operations and relations, particularly about counting up to 20, cardinality, adding, and estimation.

The teacher explained that she used state Pre-K content standards as a guide for mathematics activities; she also stated that the overarching curricular philosophy was a center-based curriculum that valued the free choice of children. The childcare center was accredited by the National Association for the Education of Young Children, and the center's educational philosophy reflected the discourse of Developmentally Appropriate Practices (DAP) (Copple & Bredekamp, 2009). The individuality of each child, the uniqueness of his or her personality, and the importance of supporting children in terms of reaching their full potential in developmental areas such as the social, emotional, physical, and cognitive domains were emphasized both on the school web-site and in personal communications with the center director and teachers. In terms of families' social class, generally speaking, they were middle class families.

Considering family engagement implementations in the childcare center, communications focused on the premise of informing parents about their children's daily lives at the center. Bulletin boards located in front of the classrooms were updated regularly to inform parents about the daily, detailed schedules of the children. Monthly parent newsletters and a quarterly journal were shared with parents about the general announcements such as field trips, guests in the classroom, and general programs of the center. Parents were invited to have daily contacts with teachers. During the research period, a parent meeting about the transition to kindergarten, and individual conferences among teachers and the parents of a particular child about the child's general well-being were held. Along with a number of parents who visited the classroom to read picture books, some parents accompanied field trips.

Participants

The Pre-K class served twenty students, eight females and twelve males. Ten of the children were White Americans, two were African Americans, four were Korean Americans, two were Chinese Americans, and one each was Greek-Cypriot and Turkish immigrants, characterizing this classroom as multicultural. For most of the children in the classroom, either English was their primary language or they were bilingual and fluent in speaking English. Along with examining the classroom as a whole, we focused particularly on the voices and actions of four children, namely Aeron, Amy, Clayton, and Octavia (pseudonyms), and their families. Since we had parental permission from twelve children's parents out of twenty and were interested in the video-recording of the mathematical practices and then, interviewing families, at the beginning I did not limit observations to any child or group of children in particular. Because of children's young age and social status, informed consent is always a process that requires attentiveness to the context and respect for each individual child (Freeman & Mathison, 2009). Therefore, once I became known to the children through my repeated visits for classroom observations and video-recordings, certain children volunteered and seemed more comfortable to be video recorded, when I asked their verbal consent to participate. Then, we used "information-oriented selection" (Flyvbjerg, 2011, p. 307) with the expectation of selecting children who might generate rich-data in terms of including "maximum variation" (Flyvbjerg, 2011, p. 307), in this case, various mathematical engagements. Among the children who seemed comfortable to be recorded, we selected four children (two males and two females) based on their diverse engagements in mathematics. As also acknowledged by the classroom teacher, generally speaking, these four children's levels and types of engagement in mathematics were different. For example, one of the boys was generally engaging in mathematically rich play in peer groups, while the other child was always playing with math materials by himself at the math corner. Similarly, one of the girls was using a

mathematically rich conversation during her games with peers, the other child was more active in mathematics activities especially in the mathematical tasks on paper. These main observations about the particular children also appeared and were confirmed during the family interviews while families were sharing their children's ways of engagements in mathematics both in the classroom and in their daily lives in general. Table 2.1 introduces the children and their families through information generated both in family interviews and my own observations in the classroom.

Table 1. Information on the Participants

Focal Children	*Family Members who volunteered for the interview*
Aeron was a four-year-old white American child, who mostly preferred solitary play by himself at the math center, particularly during free play periods. He mostly remained silent but actively completed the mathematical tasks during small group and large group teacher-directed activities.	Aeron's father was a single father in his late 20s. He explained his own previous mathematical engagements as unpleasant ones, and explained how he had difficulty in algebra during college. He stated that he supported Aeron's mathematics, especially number sense and basic number operations, particularly because he did not want Aeron to have difficulty in kindergarten.
Amy was a Chinese-American child who was four-and-a-half-years-old. She generated mathematically rich conversations with teachers and peers, and showed enthusiasm in answering teachers' mathematics-related questions.	Amy's mother was in her late 30s and she lived with Amy since her husband lived in another city because of his work. She had a graduate degree in mathematics and statistics, clearly stated the importance of mathematics in Amy's present and future life, and told stories about how she integrated mathematical thinking in their daily life routines, especially in the area of number sense, and she also engaged Amy in thinking about basic probability tasks.
Clayton was a four-and-a-half-year-old, white American child who was physically very active in the classroom and took instrumental roles. He enjoyed engaging in group play, especially with his male peers, and explicitly engaged in mathematical tasks elicited in the classroom by the teacher.	Clayton's mother and father were in their early 30s and they both participated in the interview. They described themselves as language persons rather than math persons. In many of the stories they told about Clayton's mathematics, they emphasized how he was eager to work on mathematical tasks, especially word problems, and perceived them as challenges that he needed to overcome.
Octavia was a Chinese-American child who was four-and-a-half-years-old. She remained silent in teacher-directed mathematical activities, but took instrumental roles in her mathematically rich play with her peers, especially in the outdoor area.	Octavia's mother and father were in their late 20s and they both participated in the interviews. They described themselves as art and language people, more than mathematics. They acknowledged the importance of mathematics in Octavia's future life, and believed that Octavia was very interested in mathematical learning.

Data Collection

Although the primary data collection method of this research was multivocal video-cued interviews, I started the research with participant observations to be familiar with classroom culture (Erickson, 1984) and establish a relationship both with teachers and children. After a month of participant observation, I made a transition to video-recordings in the classroom. During the whole research process, I kept field notes on a day-to-day basis, which included both descriptive accounts of participants' practices and my personal experiences such as feelings, ideas, and reactions (Emerson, Fretz, & Shaw, 2001).

In this study, I recorded videos of Aeron, Amy, Clayton, and Octavia when they engaged in diverse mathematical practices in the classroom, three days in each week regularly over a semester.

Relying on the notion that mathematical engagements are manifested at any time in a young child's daily school life (Seo & Ginsburg, 2004), I sought evidences of mathematics engagements in the classroom- through the design of the environment, and through teacher and child talk, either in teacher-directed activities or children's self-directed engagements. I recorded each of the four focal children on a day-long basis as they shifted from one context to another (e.g., small group - large group activities, structured teacher-directed mathematics activities- free play; indoor-outdoor; individual mathematical engagements-collective mathematical engagements). Then,at the end of the research process, I edited videos for each particular child down to a 10-minute video with the goal of making these short excerpts representative of the child's mathematical engagements over the three months. The particular scenes in the 10 minute excerpts were composed of three main routines in the classroom such as (a) child engaging in a mathematically rich play activity or conversations with peers (such as during block play, board games, playing cards, or during snack time, or outdoor play), (b) child engaging in mathematically rich engagements by him/herself (such as completing puzzles, playing at math corners with small cubes, sorting materials, working on number sense cards prepared by teacher), and (c) child participating in mathematics activities brought by the teacher, mostly occurring in teacher-directed large group or small group activities.

Later on, I invited families of these four children for individual interviews and used the 10-minute excerpts to elicit conversations with them. At the beginning of the interview, families watched the videos of their own child, and they had the flexibility to re-watch some scenes, or stop and reflect on some scenes. Grounded in the idea of parents as intellectual resources (Civil & Andrade, 2003), we structured interviews in a way that would allow a two-way communication between parents and me. In Spring 2012, we conducted the pilot study of this research with three parents to understand the effectiveness of the method in conducting mathematical dialogs with parents through video-cued interviews. After reviewing these previous three parental interviews, we revised and developed the latest version of the interview questions with the supervisor of this research (second author), who has experience in mathematics research with families. The interview questions were designed in a way to learn how parents' view early childhood mathematics, how they engage in mathematics with their children and how they interpret their children's video-scenes from the classroom (see the Appendix 1 Interview Guide).

During the interviews of the present study, I also shared my own ideas about the scenes from the Pre-K class and about children' mathematics in general by sharing some resources with families (The book *Family Math with Young Children* by Grace Coates, Jean Kerr Stenmark, and Brian Gothberg; and mathematics family card sets about number recognition and basic operations). Families reflected on the scenes from the classroom and also shared their experiences in mathematics with their children in out-of-school contexts. I audio-recorded all four interviews which lasted around one-hour, and then, I transcribed them verbatim.

Data Analysis

We analyzed the data in an attempt to better understand the parental insights on young children's mathematics education within the context of video-cued multivocal interviews. We approached the transcribed interview data inductively using both categorizing data analytic approaches and connecting ones (Maxwell & Miller, 2008). First, we identified codes and categories and then, developed them into three district but interrelated themes (see Figure 2.1). For example, some personal characteristics such as "shy" and "assertive" came up frequently in the interviews; therefore, we labeled them as codes. We developed all such codes into the category of "Child's Personality". As these parents' were talking about the fact that their children's personality affect the mathematical learning environment, similarly they were also talking about the "language issues" affecting the mathematical learning environment. Therefore, by using a connecting approach among all the categories affecting the children's mathematical learning environment, we created a broader theme " Context of Early Mathematics" (See Figure 2.1.). During the analysis process, notions of parents about early childhood mathematics expressed in their actual words assisted us to locate their

voices and stories around the identified themes. The analysis highlighted three distinct but interrelated categories such as parental insights on (a) the content of early childhood mathematics (b) pedagogical ways of teaching early childhood mathematics and (c) the context of early childhood mathematics.

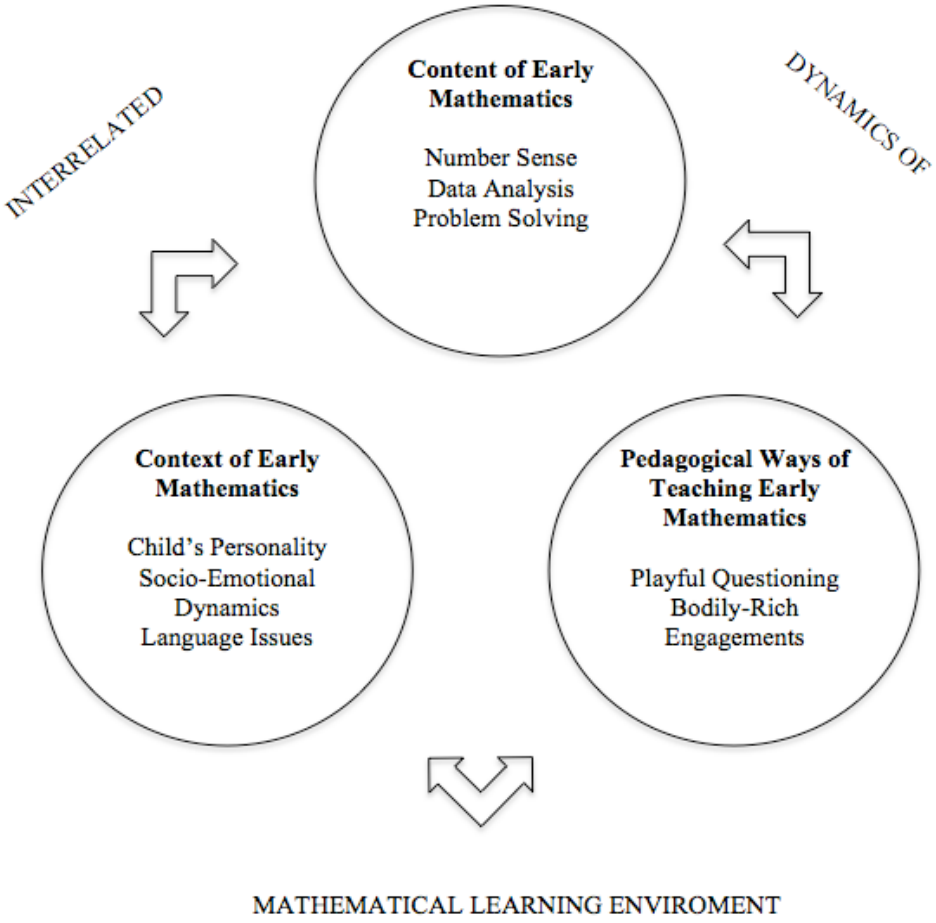

Figure 2. Parental Insights on Young Children's Mathematics

Findings

Parental Insights on the Content of Early Childhood Mathematics

In the interviews, all parents shared various foundational aspects of mathematical content that they were drawing on when engaging in interactions that involve mathematics. These interactions are either initiated by children or by their parents when they mathematize children's daily practices, with examples such as corresponding the number of family members with the food bought at the grocery store, or with the utensils on the dinner table. Not surprisingly, the major theme in mathematics content shared by parents is numeracy, which includes both number relations and number operations. It is well stated in the literature that parents report mostly numeracy activities as part of their children's mathematics (Blevins-Knabe & Musun-Miller, 1996; Skwachuk, 2009). The type and the nature of these numeracy activities, however, have not been articulated clearly and in detail in previous research. Parental dialogs reported in this study manifested the ways these four-year-old children engaged in different dimensions of numeracy concepts. Parents, for example, reflected that their children show understanding of cardinality and engage in skip counting by 2 and 5, which are the practices that they engage in at the Pre-K classroom.

Besides these basic numeracy concepts, Octavia's and Clayton's fathers, both reflected that as their children count, they show interest in the ascending nature of numbers, and show excitement and curiosity when talking about big numbers. After watching the video of Octavia answering the teacher's question about how many Easter eggs she collected by replying as "one trillion", her father shared:

> *I think she has a good sense of numbers. She says one trillion, probably she has no idea how large the number is, but she knows that it's a very large number. So, she expresses certain meanings in numbers. Yeah, she is trying to exaggerate [referring to the video scene]; sometimes at home she uses the word infinity. She knows that was not the correct answer, but just exaggerates like "infinity!". She made up the words like cazellion, bizillion, gazellion. So, I do not want to discourage her. I say, "Ok yes, it is a word, then." She asks how large is a number. I say, "I do not know. Almost infinity…"*

This experience shared by Octavia's father shows how he scaffolds Octavia's interest in big numbers, which is framed in imaginary words, through introducing the concept of infinity appropriately in response to her questions. Preschoolers' intuitions about the concept of infinity is acknowledged in previous studies and these studies suggest a similar approach in responding to children's questions related to the ascending nature of numbers by clarifying the fact that there is no greatest number (Lappan & Wheeler, 1987). Likewise, in her study, Evans (1983) exemplifies how the integration of infinity and zero concepts, perceived as impressive by kindergarteners and first graders, enhance their mathematical thought, which is evident in their problem solving skills. Although it may be developmentally early for four-year-old children to engage in thinking about these concepts rigorously, it seems to be effective to positively guide them and foster their curiosity when children initiate these conversations, as modeled by Octavia's father. Similarly, focusing on the concept of zero, Clayton's father shared that:

> *Recently he wanted us to quiz him in math… So we sit there, and he says "Give me a math problem". Ok, so we do like what's 2+3, what's 4+3 and he has figured out connections (among numbers) so he says his sister is zero and he is five; that when he is six, sister will be one … He'll say "daddy, daddy when I'm 20, how old my cousin be" and … he does these on his own.*

In this example, based on his father's sharing, Clayton shows an emerging understanding that zero has a numerical value, which is smaller than 1. In an imaginative mental number line, while he is moving to be six, his sister will move to 1. Clayton's understanding of zero is in line with the research suggesting that toward the end of the preschool years, children pass through steps related to the understanding of the zero concept, starting from recognizing zero as nothing and then, discovering its relationship with other numbers on the number continuum (Wellman & Miller, 1986). This ability of Pre-K children to recognize zero as an ordinal property in a numerical continuum sets the foundation of their future algebraic understanding (Meritt & Brannon, 2013). Clayton's and Octavia's fathers help us to see how these young children's interests in the concepts such as zero and infinity hold potential for their further understandings in numeracy and algebraic concepts. These careful observations shared by parents suggest that preschool teachers could be encouraged to integrate meaningful conversations with children about these concepts in the curriculum through daily conversations (Lappan & Wheeler, 1987). Teachers could learn to guide children in a way that would not cause misconceptions when opportunities present themselves. Among the diverse ways to support children naturally can be counting backward until zero through songs, and rhymes, and including the written symbol of zero during mathematics activities (Wellman & Miller, 1986), associating "nothing" conceptually with the zero, and forming up creative questions that would lend themselves to considering the infinite nature of numbers.

Another theme on mathematical content shared by all parents is related to data analysis, a skill that requires other foundational mathematical knowledge such as number sense, comparison, categorization, and patterns. The dialogs around data analysis presented examples of children's transference of some mathematical practices from school to home. Focusing on mathematical practices in the Pre-K class, in specific cases, they have a weather calendar in which they record the number of sunny, rainy, and windy days in a month. Similarly, as mentioned earlier, they do voting charts on two or more choices to make democratic decisions as a classroom community. Both Amy's and Octavia's mother touched upon how their children reproduce similar charts at home, and explicitly reflect on what their children have been crafting mathematically through these charts. Amy's mother, who has a solid background in statistics and mathematics, shares that:

> *The other day I saw the chart (a voting chart with two columns showing the friends who wanted and didn't want to participate in physical education class. The columns have the names of the friends and the bottom line was showing the numbers of friends in two columns). You actually can see the frequency; it's like a frequency chart, and I told her that and we analyzed the data, so Amy draws something like those…"? … "We play together, and she can show like the bar graphs. She can draw the pie chart, and she has not any problem to draw those.*

The example provided by Amy's mother shows how she is recognizing and supporting her child's mathematical practice by using a language that has a rich mathematical value such as framing the choice of attending physical education class as a frequent choice, and the number of peers who voted as data. Drawing on the idea of parents as intellectual resources (Civil & Andrade, 2003), her recognition of Amy's chart as a frequency table and how frequency tables can be represented through bar graphs and pie charts, which have a potential for data analysis, affirm that parents bring their own mathematical backgrounds to their interactions with children and also engage in interactions that have potential to move beyond the classroom practice regarding the mathematical content. Octavia's mother (who described herself as an "arts and language person") also reflected on a similar data analysis example. After seeing a video scene where children were engaging in completing the weather calendar, she stopped the video and shared a photo she took previously, showing Octavia producing a chart at home. The chart had four columns for each season, and several dots in each column under the season name.

> *Mother: She drew this (takes her phone and shows the photo to father and the researcher). We were not noticing that she would draw this. This is the fall, winter, spring, summer (points the columns photo). I asked what those dots meant and she said the person numbers that liked it (number of people, represented in dots, who liked the specific season). But, it's not actually 12 dots.*
>
> *Researcher: She is kind of imitating. You know, what they are doing in the class is very similar to this. She is reproducing in a very neat way. Actually, almost exactly how teacher is showing….*
>
> *Mother: Yeah, I'm sure. She also writes like less, and more. She compares which has the most number and which has the least number. 0 is corresponding less and 14 is more (points the chart).*

This engagement of Octavia and the way Octavia's mother talks about her observation highlight the rationale behind this mathematics activity, first conducted in the classroom and then reproduced by the child at home. The data analysis engagements at home are rich and linked to children's understandings of number sense, representation, classification and categorization (Cross et al., 2009). Octavia shows understanding of number relations including the concept of zero; and symbolically represents the peers' names in dots, and then, in numbers, which is an intricate skill for a Pre-K child. Octavia's mother recognized, kept a record of this engagement, and shared this during our dialog. In other words, she assisted us to see how young children reinforce and practice their own particular mathematical learnings across different contexts.

Among the other types of mathematical content shared by parents were basic number problems, both in algorithm and story forms. Interestingly, basic geometry, considered as a core area in early childhood mathematics (Arnas-Aktaş & Aslan, 2005) did not manifest itself during the interviews, which is also an area the teacher was not focusing on during mathematical activities in the classroom. Overall, these dialogs with parents highlighted mathematical content knowledge their children are engaging with in out-of-school contexts as well as the way parents support the learning of this content knowledge. The diverse ways of parents supporting the mathematical learnings of children will be broadly analyzed subsequently under the category of pedagogical ways of teaching early childhood mathematics.

Parental Insights on the Pedagogical Ways of Teaching Early Childhood Mathematics

In looking into the stories told by parents we can witness the diverse methods of teaching that parents are drawing on in their mathematical engagements with their young children. Appropriate to the ages of their children, playfulness and pleasure is integrated as parents attempt to teach mathematics to their children, or as they responsively react to the mathematical opportunities brought by their children. Parents reported a playful response particularly in the child-initiated dialogs composed of formulating and answering mathematical questions.

In their dialogs, parents brought out the idea that their children are spontaneously engaging in mathematics in daily life (Ginsburg, Inoue, & Seo, 1999; Seo & Ginsburg, 2004). Parents' recounting of their dialogs points out that they recognize natural learning moments, and they are building time and space to further enrich those experiences without directly explicating them as mathematics. Both Aeron's and Clayton's fathers, for example shared examples of these everyday moments. Aeron's father, who plays guitar and studies music production, frames "*the drums as the most mathematical instrument*" and shares his observations in relation to use of the body, music, and mathematics:

> *Father: He [Aeron] likes drums, so he likes to play, kind of like along the beats, with me. He is actually a good singer too, but he won't do it in front of everybody, only me [laughs]… Yeah, he is shy.. Umm, but his favorite is the drums, he loves the drums.*
>
> *The researcher: So, I think there is some connections with mathematics and drums.*
>
> *Father: Oh yeah, oh yeah, I mean you have, it's like the main beat of every song, you have to count the beat while you're playing. You know [father taps his feet]. It's very, probably, the [drum], the most mathematical instrument. He like, I gotta couple of little drums for him, he really likes them a lot too, but you know, just, he can keep the rhythm, you know. And that amazes me, that a 4-year-old, he can keep the rhythm. Well, he uses his hands for that, and like it's not like he's counting the beat yet, like a musician would do, but he has, has the feel of it in his hands, like he can feel the rhythm and his hands just do it, you know.*

Aeron's father is exemplifying the research that has been informing us about the mathematical opportunities embedded in musical activities for young children. Through appropriate scaffolding, integration of music and mathematics can add playfulness, pleasure, and meaning to mathematical practices (Lim-Kim, 1999) and has potential to afford enhanced understandings in geometry, number sense, and measurement (Benes-Lafferty, 1995). Interestingly, a very similar observation was shared by Clayton's father at the very beginning of our video-cued interview, when he was introducing himself:

> *…Math has played a huge role in our extracurricular activities and she [Clayton's mother] did dance. There is a lot of rhythm and counting and timing with that, knowing when things are coming, sequence. And that same thing, I did music and I think Clayton has a good portion of both of that… that he expresses himself with all parts of his feet, his hand, his arms and he's very much in rhythm. It's impressive, for even as far back [as when] he was 2 or 3 [years-old], he was nodding his head in time to the music and now, you know, he'll kind of play the drums and drum my lap do some. And I do things [that] aren't so much straightforward, you know, and he'll keep up with that. And so he's got a good internal clock, you know, and rhythm…*

Later on, Clayton's father continued with a similar theme, when he was giving an example from his own childhood related to mathematical engagements:

> *Father: I remember music class, like first learning time signatures, that a half note is two, and a quarter note is one. You know, they [teachers] pass all the wood blocks and triangles.*
>
> *Mother: And he does that too, he is in choir. They have their rhythm sticks and they so yeah, he is doing that. Clayton is doing that too [in choir].*
>
> *Father: I think it would be easier if you told him, you know, dance out, stamp seven times or hit a wood block or triangle seven times, than say count to seven. That would be easier for him to do that.*

It is interesting to see how parents' observations are parallel with the current literature on understanding the role of bodily movements in acquiring mathematics. From an embodied cognition perspective (Núnez, Edwards, & Matos, 1999), attention to children's bodily experiences is recognized as important in their mathematics education. A growing body of research shows that particularly rich bodily experiences are central to children's mathematical thinking (Alibali & Nathan, 2012; Valenzeno, Alibali, & Klatzky, 2003; Kim, Roth, Tom, 2009). Researchers are proposing that children learn mathematics not only through their bodily experiences but they are also thinking through their bodies.

In their dialogs, both Octavia's and Amy's mothers shared their observations about the way their children were creatively using their hands and toes for counting, and feet for estimation and measurement. Overall, although we do not know to what extent and in which ways these parents were explicitly mathematizing these moments and appropriately scaffolding for their children, we can see that they are aware of these teachable moments and recognize them as rich opportunities for their children' mathematics. This is a promising awareness on parents' part and shows the potential to collaborate with parents to draw on these integrated everyday moments as mathematically resourceful ones for children. The Family Math for Young Children (FYMC) program, for example, could be a resource for this collaboration, in which, families engage in combining movement and mathematical experiences to enhance children's understudying of mathematical ideas (Dávila-Coates & Franco, 1999).

Another theme that came up under the theme of parents' pedagogical ways of teaching mathematics, is questioning. During the video-cued interviews, all parents shared stories in which their children are initiating conversations full of formulating and answering mathematical questions. Parents were clearly articulating the phenomenon of their children's asking mathematical questions at home, and also the way that they shape these questioning moments as appealing ones for their children by encouraging them to continue to do so. For example, Octavia's mother and father shared that:

> *Mother: [Octavia says] "Mom, do you know what 200+200 is, do you know what 1 zillion + 1 zillion is?" Always those kind of questions she would ask, [then] she would, proudly [say], "I know the answer!" I would pretend, "Ohh, that's too hard! Then, she will very proudly say, "That's 400!, and do you know 8+8?" And, I, "Umm, let me think," and she would say like "16!" She always gave us math quizzes at home, and we encouraged those, because we always pretend, like, "Umm, that's difficult, let me think…" Then, she has the, has the feeling…*
>
> *Father: Yeah, we would intentionally give the wrong answer, then she would correct us.*

Children's mathematical understandings can be fostered through interactions including a diverse variety of questions, but only if these questions are in line with what children already know and are capable of accomplishing; and only if they are interesting ones (New Zealand Maths, 2011). Octavia's mother's and father's engagement shows that parents are making this process interesting for their children by reacting responsively and adding playfulness to the questioning process. Clayton's father also illustrates this process initiated by Clayton:

> *He asks the questions that he really he wants to answer. So, he'll say, "Daddy, daddy, when I'm 20 how old will you be?" And, we say, "I don't know buddy, you tell me cause you know their ages, so you figure out." So, he thinks about two minutes and he finally does it.*

In this case, Clayton's father shows how he is encouraging Clayton to solve the mathematical problems that he is formulating by himself. The rationale behind this questioning is interesting. The father realizes that Clayton is capable of solving the problem, but that sometimes he is asking these questions to show his own mathematical understanding to his parents. Similarly, when asking mathematical questions to her parents, and then proudly answering them by explicitly saying that she knows the answers, Octavia is showing the same intention in her engagement.

> *Amy's mother: We're writing [numbers] and we have an abacus, and she also solves the problem using [the] abacus. For simple ones, she uses her fingers, and if fingers are not enough, then, she uses toes, so 20 for fingers and toes. She can do like that. Then, umm, later, I ask and write problems and we can solve on the paper. And she can do [it] very quickly and she sometimes makes problems by herself. She just prepares the problem and she pretends as if she is the teacher and she solves the problem. She does that.*

Amy's mother allows us to see how she is participating in Amy's imaginative play in which Amy is acting as a teacher, again with the similar aim to show her own understanding to her mother. Parental dialogs allow us to see how Pre-K children are aware of their mathematical learnings and enthusiastic about showing these understandings. Thoughtful questioning is a way to support children's mathematical understanding. In the process of questioning, while inexplicit open questions allow children to think flexibility and come up with creative answers, explicit-focused questions, similar to the ones engaged in playfully by parents and children in this study, keep children focused and encouraged (Parks, 2010). These engagements of parents and the way they share their experiences hold potential for us as educators to communicate and collaborate with them deeply in terms of generating a wide repertoire of questions that would strengthen children's mathematical thinking.

Parental Insights on the Context of Early Childhood Mathematics

Listening to parents' voices through video-cued interviews allowed us to see that in their close observations parents are going beyond individual mathematical learning, content, and mathematical tasks; they are showing awareness of the children's learning context broadly. Parents see the mathematics of their children, yet they also see the dynamics of the whole learning process. Grounded in the notion of parents as intellectuals (Civil & Andrade, 2003), and drawing particularly on the *parents-as-parents* conceptualization, we could see that along with the close observations, parents bring their own sensitivity as parents to the understanding of children's mathematics. In other words, dialogs with parents yielded insights about the affective component of the mathematics learning process (Civil & Bernier, 2003). The dialogs with parents illustrated how they are focusing an eye on their children's social interactions with others during the learning process, including conflicts, negotiations, and in terms of showing respect and caring for others. Moreover, parents of the bilingual children Octavia and Amy, were concerned with their children's emotions resulting from their possible struggles, particularly because of the language issues they imagined their daughters might encounter in the context of mathematics. These diverse context-related sensitivities were evident in the stories told by parents. In Clayton's mother's words, "these are the side stories but math stories".

Clayton's Mother: …Last time, ok, this is a side story but it's a math story… so we're lying back last night and he said, "Can we do one math problem before I go to the sleep?" "Yeah, I said sure. What's 6 plus 2?" And he said, "Ohh, that is easy Mom, that's eight," and I said, "Good, ok." And I said, "By the way, what was your small group today?" He said, "umm, it was iPad … me and a friend, we're doing dominos …. we had to match the number with the dots." And I said, "Wow, was it easy? He said, "Yeah, it was easy for me, but it wasn't so easy for my friend." And, "Yeah," I said, "Did you help him or did you boss him?" He said, "No, I helped him," and I said, "Was [it] that he does not know numbers?" And he said, "Well, he knows the easy ones, he knows 1,2,3,4,5, but he doesn't know the hard ones like, you know 7,8,9,10 and a hundred " "Yeah," and I said, "yeah, ok…."

As introduced before, Clayton is a physically active child, who mostly engages in social interactions with both friends and teachers. He is clear and explicit when talking about his ideas, and he actively engages in classroom discourse in general. As his parents describe Clayton as having high self-confidence, they are sensitive about whether he is being assertive and interrupting both others' and his own learning process. Later on, as we were watching the video in which Clayton was playing chess with a friend, this time his father made the same observation. In the video scene, Clayton was reminding his friend, with whom he was playing chess, that he should do his moves quicker as they were taking turns.

Clayton's father: This is what Angelina was saying, did you boss them? Did you help them or did you boss them?

Clayton's mother: A little bit bossy…? Umm, Onur, is like his brother. If it was [the] other kid, I would kinda feel bad…

As pointed out by Clayton's parents, social interaction in mathematics learning is a constantly reciprocal and dynamic process between and among individuals. This dynamic nature of interaction also results from the individuals' uniqueness because "each person is the unique product of a range of socio-historical cultural communities and practices, of unconscious drives and desires, as well as propensities by virtue of genetic make-up and socio-cultural location" (Lerman, 2001, p.94). Focusing on Aeron, who is a more silent child, who mostly engages in solitary play, and prefers playing at the math corner during free play time, but mostly by himself, his father also shared concern about social interactions over a video scene focusing on mathematics. In the scene, after playing with a material that has mathematics value, Aeron attempts to join another friend's puzzle game, and at first he is implicitly rejected.

Aeron's father: That's pretty cool…I like that (a play material for matching object pictures with the objects' initials). Sometimes, he is not good at speaking about his feelings or telling someone that, you know, telling on someone, you know like, I was playing with that, nobody, he didn't ask, you know, like, he just sometimes, he just would rather let it go than to speak up about it, you know…

These parents are highlighting the need of acknowledging individual needs of children, and creating an environment where their personality characteristics would not prevent them from engaging in classroom learning activities and resources. Conceptualizing learning as a process of participation in communities of practice (Lave & Wenger, 1991), each community such as a class, family, or specific culture has its own unique practices including language, values, identities, artifacts, norms, and rules. Lave and Wenger (1991) emphasize the importance of the critical issues learners face during this process. Power relationships and hegemony over the resources of the community affect how learners shape their identities in the community and how they are encouraged in, promoted in, or alienated from the community. Focusing specifically on language dimension, Octavia and Amy's mothers shared their concern about the fact that their children's home language is different than the academic language in schools, including in the area of mathematics:

Octavia's mother: Right now, like my vision is that she will be struggling with her math education, especially…

Researcher: Because of the language?

Octavia's mother: Yes, especially the terms (referring to mathematical terms)… So, I do plan to teach her in the future like that my understanding in some Chinese ways...

Researcher: This would be a really rich experience. Language and thinking, they have very strong connections, so if she can study mathematics both in Chinese and English, that would be really good for her cognitive thinking.

Octavia's mother: Yes, yes.

Octavia's father: …and math itself is a language, so it's just the foreign language, it's not a linguistic language but it's a way of thinking…

Although Octavia is a child with intricate mathematical skills, as acknowledged by both her parents and the teacher, as a parent her mother is concerned with her mathematical development, and feels the need to support her child in her native language. Interestingly, both Octavia's mother and father emphasize the relationship between language and mathematics by situating mathematics as a special language by itself with its special terms. In other words, these parents are showing awareness of how mathematical literacy is different from the daily language their bilingual children are engaging in, both in- and out-of-school contexts. The concern is reasonable as evidenced by previous studies pointing out the struggles of diverse children and families with mathematical literacy (Martin, 2006) and with their participation in mathematics. From a pedagogical perspective, Octavia's mother is explicit about supporting her child's mathematics in "Chinese ways." When she is referring to "Chinese ways," she is implying the Chinese language, but also certain ways of teaching mathematics in Chinese educational culture. Amy's mother exemplifies these methods in detail:

Amy's mother: I think she is talented, she'll probably do mathematics better than me… But, (it) depends on… I'm not familiar with the American system, how they train those kids and so, I will try very hard at home, to make her interested in math. We have, for example, multiplication, I'm not sure how here they teach multiplication. In China, we have, like a table and so you can memorize very easily, and you can do like, from one times one to nine times very easily, and for me it works very well. But I'm not sure how that exactly is done in English, because if you translate in English it would have actually… If you speak Chinese, you can easily memorize it, but Amy is thinking differently than me, because I'm familiar with Chinese and she is not, so she actually didn't get that part of… I can do it, but I try to teach her this, she is less interested in [learning the Chinese way of mathematics] because of the language….. for me (referring to her own childhood) it would be very hard to count in English, as well.

Amy's mother is touching upon the idea that language and mathematical concepts have strong connections, and language has power to shape the conceptualizations of mathematical concepts. As pointed out by Amy's mother, Chinese and English languages differ in terms of naming the numbers words, which have implications in learning the base-ten system (Cross et al., 2009). Similar to Octavia's mother, Amy's mother is also planning to support her daughter's mathematics both in English and Chinese by drawing on the methods that have been used in Chinese education, which has been shown to be effective in supporting young children's academic achievement, including the area of mathematics (Barnett, Yarosz, Thomas, Jung, & Blanco, 2007).

Conclusion and Educational Implications

"Learning must be understood with respect to practice as a whole, with its multiplicity of relations - both within community and with the world at large (Lave & Wenger, 1991, p.141). It's clear that it is the nature of social interaction that shapes and is being shaped by the multiple identities of each unique individual in the learning process as a whole, including the mathematically rich learning opportunities residing in these social interactions as the focus of this research. Parents have a unique privilege in observing and interacting with their children; their insights and experiences of the mathematical learning process have implications for both educators and educational researchers in terms of what is valuable to teach children, what ways are meaningful for creating learning opportunities, and what are the questions that we are searching answers to in the hopes of a constructive change in early childhood mathematics education.

The dialogs I engaged in with parents drew a broad picture of the landscape of early childhood mathematics, as we elaborated their notions and observations about content in early childhood mathematics, particularly parents' observations of diverse number sense and data analysis engagements with their children. We also saw parents' pedagogical ways of teaching mathematics, particularly in their use of questioning and the integration of play and movement. Finally, we heard parents' reflections on the context of mathematics in which parents shared their deep concerns and sensitivities about the processes of their children engaging with mathematics.

In looking into the family engagement literature, there is consensus that parental engagement is necessary and ultimately good for children's academic content areas including mathematics; however; the definition exactly what we intend for this engagement is vague and it is not commonly acknowledged that the phenomenon of parental engagement is not pleasurable for all parents (Dehli, 2003). We should be critical of the common assumptions that only expect families to support their children's pedagogic practices in line with the school practices, which are for the most part disconnected from their home experiences. Research conducted from this frame of mind can lead us to the conclusions that would position parents as responsible for their children's lack of mathematics knowledge while also positioning parents as being in need of learning mathematics to be able to support their children's mathematics. This perspective lends itself to a promotion of general parental engagement practices with the expectation those will be fit for all parents and all children's needs. However, "parents are not participating in their children's education because they believe they ought to be or because some is telling them to, they do so because they believe they must in order the produce the effects they hope for" (Freeman, 2008, p. 472). When designing the research we reported here, this perspective allowed me to move away from concerns about whether or not parents are capable to support their children's mathematics; or concerns about whether or not they have enough means to stimulate their children's mathematical interests at home or if they value language and literacy experiences over the mathematical ones during the early childhood education period. Our aim is not to ignore these concerns, because they have emerged frequently in research contexts where understanding the process of children's mathematics learning is a focus; instead, our goal is to move these concerns away from the center of the inquiry. The research we reported here based upon this insight made it possible for us to see that as long as we develop a methodology that allows us to draw on the strengths of parents, we could see that the notions coming from parents are powerful and very in line with the research exploring young children's mathematics education broadly.

Cross, Woods, and Schweingruber (2009) acknowledges the effectiveness of programs that link families and schools in terms of enhancing children's mathematics; however, their statement confirms our lack of knowledge about particular effective practices for educational programs to support parents and young children in mathematics. The results of this video-cued study with parents have potential to inform practice in terms of first acknowledging parental knowledge and experiences as a source of knowledge for early childhood mathematics as a starting point, and then, creating

opportunities where teachers and parents might come together and engage in thinking and learning from each other in terms of children's diverse mathematical engagements. Practices that already widely exist in schools, such as family math nights and school family math clubs, would be rich spaces for these engagements if designed from the parents-as-intellectual resources point of view. We imagine that the dialogues might take place in these spaces could expand our thinking on mathematics in the early years and could also provide us with an understanding of how we might make curricular connections with parents, as they share how children engage in and transform mathematical practices as they move back and forth between home and school. Put succinctly, we envision that efforts to improve early childhood mathematics education can be developed as a way of thinking about how to support parents and how to be supported by them in these shared spaces.

Recommendations for enhancing early childhood mathematics education regarding content focus on number sense and geometry as core areas incorporating early number operations, spatial thinking, measurement and data analysis (Cross et al., 2009) along with the "habits of mind such as curiosity, imagination, inventiveness, persistence, wiling to experiment, and sensitivity to patterns" (Clements, Sarama & DiBiase, 2004, p.57). Parents in this study discussed how they draw on and observe their children's engagements in some of these content-related areas, particularly in terms of number sense, number relations and basic number operations, and data analysis -which is rich in terms of patterning and knowledge organization. The content areas of geometry and measurement did not emerge in the research with parents, but this may have been due to the lack of these areas in the classroom video excerpts the parents observed and reflected upon, rather than being due to a lack of engagement with these areas with their children. When we take a more specific look at the areas of content parents explored with their children, we see some extensions beyond those outlined in recent literature about early childhood mathematics. Parents and children did not simply explore numbers 1-30 in the area of numbers sense, but were interested in contemplating "big numbers" such as a [gazillion]. Children did not restrict themselves to simple addition concepts such as considering how many pears and apples were in a basket-instead they wondered about how old their sister would be when they were two years older.

It is well stated in the literature that children engage in a great deal of mathematics as part of their everyday lives both in and out-of-school contexts (Seo & Gingsburg, 2004), therefore; it has been recommended that early childhood educators recognize and mathematize these everyday situations with young children through elaborating on mathematical ideas along with appropriate math talk. As parents shared their observations, not only related to their children's mathematics, but also about the very nature of mathematics itself, we could understand that they see how mathematics is embedded in their children's daily lives; and they confirm that these everyday moments are rich for mathematical engagements, particularly when children bodily engage in physical play, music, and dance. On the other hand, it is not clear to what extent parents are mathematizing those moments, and in what ways they insert themselves into these teachable moments; but this could be a starting point for further research about observing these processes and generating insights for both parents and teachers in terms of turning teachable moments into mathematically meaningful ones for children.

In addition to these inadvertent everyday learning opportunities, intentional teaching moments (Cross, et al, 2009; Ginsburg, 2009) provide a context in which children engage in a foundational mathematics content under adult guidance with a particular goal related to the mathematical engagement itself. Parental dialogs in this study showed evidence that parents balance both intended and non-intended learning moments in the course of mathematical engagements. Parents do not only engage in mathematics that appear in their children's daily lives spontaneously, but they also create intentional teaching moments or participate in mathematical practices brought by their children with a direct intention to teach mathematics such as engaging in imaginative play by taking the roles of teacher and student; or acting as if the answer of the mathematical question is not known themselves during the mathematical questioning moments.

The potential we see in these dialogues that take place between parents and researchers, and among parents and children, is that each actor, including the child, in the process will have mathematical insights to share with others. Specifically, in early childhood education, along with curricular knowledge, the idea of what is valuable in terms of mathematics would be a constructive topic in communication among teachers and parents and children. In relation to these dialogues, Walkerdine (1988) supports the notion that the discourse on mathematics changes as individuals and their discursive practices change. Therefore, when families, children, and teachers come together and engage in communication, they will produce and will be produced by a unique discourse on mathematics. Moreover, this dialogue could create a space for observing how children transform mathematical practices between home and school as parents and teachers share their own understandings of mathematics. These dialogues would expand the thinking on early childhood mathematics and also provide us with an understanding of diverse families' perspectives on mathematics. How children transform the mathematical practices between home and school and the anxieties and pleasures of this process for teachers, parents, and children seems to be important and exciting foci for future research that would enrich our current knowledge about early childhood mathematics education.

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Appendix 1. Interview Guide

"Thank you for being a volunteer in this interview. I'm a PhD student in Early Childhood Education program. As you know, I've been conducting research in your child's class. The aim of this interview is to learn your beliefs about mathematics from a parental perspective. During this process, I will audio- tape our interview for transcription purposes. I will destroy the audio after transcribing. As we talk, please feel free to ask me any questions. If there is anything you don't want to answer, you can just say so. Do you have questions before we begin? I' have the videotaped scenes of children when they are doing mathematics. I'm interested in watching it with you to hear what you think about the video."

- ***Could you please briefly introduce yourselves?***
- ***I want to start with a general question. What comes to your mind when you think about mathematics?***
- ***What is your experiences related to mathematics in general?***

As an adult; as young child?

- ***Can you tell me a scene that you were doing mathematics when you are a child? / either remembering inside or outside of the school?***

Now, I will show you the video of Olivia. As I told you I have been recording their various engagements during a day. Now, I will show you her video. Please feel free to stop the video by using space, and we can talk about some scenes if want to.

- **What do you think about your child's video in general?**
- Probes: What was most striking about the video?
- **Are you informed about the mathematics activities in your child's class?**

Probes: Do you communicate with teacher about the activities in class? Are you asking your child about the activities in the class? Does your child tell you about the school activities that may have mathematics component?

- **Do you engage in mathematical activities with your child at home?**

Probes: Do you have similar observations of your child's when/he is out of the school? (what can be considered as mathematical)
In daily life, while talking to your child, do you make connections to mathematics such as counting, matching, simple addition - subtraction? Do you provide your child materials, or toys which may have mathematical value?

- **What kind of play materials do you have at home? What do you think about their value related to mathematical thinking?**

Probes: Talk about puzzles, blocks, board games, number games.

- **I would like to continue with the idea of mathematics in early years. What comes to your mind when you think about mathematics in early childhood education?**

Probes: What kind of practices can be considered as mathematical? (For example, counting, or sorting, matching, ordering objects, or solving problems, talking about time, size etc.) Do you observe your children engaging in these practices?

- **How do you see the importance of mathematics education in the early years?** Probes: Do you think that it may impact children's future mathematical practices or their attitudes toward mathematics?
- **What do you think about the importance of mathematics in your child's life in general?** Probes: Do you consider mathematics as an important area during your child's early schooling? In what ways, are you planning to support your child in the area of mathematics?

Thank you for you participation.