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Comparison of Turkish and American Seventh Grade Mathematics Textbooks in Terms of Addition and Subtraction Operations with Integers

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Abstract

This study analyzes how addition and subtraction with integers are presented in Turkish and American mathematics textbooks. Analyses focus on how the concepts are given as well as the nature of the presented mathematical problems. It was found that both the Turkish and the American textbooks emphasized the relationships among different representations in teaching addition and subtraction with integers. It was found that the coordination among visual representation, verbal explanations and mathematical sentences was constructed in a more organized manner in the textbook named *Connected Mathematics 2*. It was found that operational skill oriented problems were proportionately featured more in the Turkish textbooks whereas the problems requiring high-level cognitive skills such as mathematical reasoning and problem posing were featured more in the American textbooks.

Keywords

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Introduction

Textbooks occupy an important place in the implementation of national curricula and the transfer of knowledge to students (Sood & Jitendra, 2007). Studies indicate that textbooks influence the types of activities teachers conduct in classes, as well as the approaches they use when presenting new concepts (Fan & Zhu, 2000; Li, Chen & An, 2009; Sun, Kulm & Capraro, 2009). Tutak and Güder (2012) determined that the large majority of fifth grade teachers use textbooks as their main sources of reference. In this context, the analysis of textbooks not only serves to demonstrate the different learning opportunities countries present to their students, but also allows the identification of similarities and differences between countries with regards to their teaching activities (Kilpatrick, Swafford & Findell, 2001; Mayer, Sims & Tajika, 1995). In addition, the results of such studies contribute to determining and explaining the reasons for the performance differences observed between countries in the Programme for International student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) program (Alajmi, 2012; Son, 2012; Son & Senk, 2010).

Although it would not be possible to demonstrate differences in student performance based solely on the analysis of textbooks, numerous studies emphasize that the analysis of textbooks nevertheless provide an important tool and mean for explaining the performance level of students

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(Alajmi, 2012; Li, 2000; Mesa, 2004; Sonk & Senk, 2010; Valverde, Bianchi, Schmidt & Houang, 2002). Tyson and Woodward (1989) described that 75-90% of the time spent on teaching in the United States is organized around textbooks. Evaluations performed on classes in Norway, Spain and the United States demonstrated that teaching processes remain largely dependent on textbooks, and that, as students progress towards higher classes, teachers become even more dependent on textbooks (Schmidt, McKnight, Valverde, Houang & Wiley, 1997). In Turkey, textbooks are heavily used especially in elementary and middle schools (EARGED, 2003), while high schools tend to make greater use of test books and supportive books for university preparation (Altun, Arslan & Yazgan, 2004). According to the 2007 report of TIMSS and the 2006 and 2009 reports of PISA, American students demonstrate higher performance than Turkish students. In this context, this study aimed to compare American textbooks that have adopted the principles and standards of the National Council of Teachers of Mathematics [NCTM] (2000) with Turkish textbooks prepared according to the 6th-8th Grade Mathematics Program introduced in 2005 in terms of how they teach addition and subtraction with integers, and the types of mathematical problems they present. The results of this study may contribute to explaining the performance differences observed between these two countries in comparative international studies.

In this study, the comparison of textbooks was performed based on addition and subtraction with integers. Integers and operations with integers have an important role in the learning of more advanced subjects such as algebra (Gallardo, 2002; NCTM, 2000; Vlassis, 2004). Despite this importance, studies indicate that students or teacher candidates experience difficulties in operations with integers (Avcu & Durmaz, 2011; Türnüklü & Yeşildere, 2007). These difficulties may stem from issues relating to teaching activities on addition and subtraction with integers. Consequently, it becomes important to analyze the type of learning opportunities countries present to their students during the teaching of integers. Considering that textbooks are important materials for predicting student performance, the analysis of textbooks may provide important cues regarding the teaching opportunities presented to students with regard to the learning of integers.

Steiner (2009) described that there are two models used in the teaching of integers. These models are the number line model, which emphasizes ordinality; and the neutralization model, which emphasizes cardinality. Steiner also described that teaching activities which combine ordinality and cardinality have a positive effect on conceptual learning. Similarly, Van de Walle (2007) recommended the inclusion of activities related to these models in order to ensure that students gain a better understanding of integer operations. For neutralization model, examples from daily life such as profit and loss, hot and cold should be used. Altınok, Keşan and Yılmaz (2005) described that the teaching of integers by making associations with daily life has a positive effect on the learning of students. Therefore, the analysis of textbooks can provide important cues on how operations with integers are associated with visual models and daily life situations.

Studies on the comparison of textbooks are performed based on the physical characteristics of the textbooks, the presentation of concepts, and the characteristics of the mathematical problems. Erbaş, Alacacı and Bulut (2012) previously performed a study investigating the physical characteristics of sixth grade mathematics textbooks from Turkey, Singapore and the US. The textbooks were analyzed with respect to their visual design, their subject intensity, the sequence between their subjects, the subjects they covered, and the approach they used for presenting their subjects. Arslan and Özpınar (2009) analyzed sixth grade mathematics textbooks according to the views and opinions of teachers. In their study, they determined that textbooks did not take into account the preliminary knowledge of the students; that the relations they established between different subjects were inadequate; and that the assessment questions were limited to practice questions. In literature, the emphasis is mainly placed on analyzing how textbooks present concepts, and on determining the types of mathematical problems they contain (Alajmi, 2012; Mayer, Sims & Tajika, 1995; Son, 2005, 2012; Son & Senk, 2010). Aktaş and Aktaş (2012) investigated how the relations between quadrilaterals were explained in the seventh grade elementary school mathematics curriculum, textbooks and student workbooks. They concluded that a systematic method was not used in explaining the relations between quadrilaterals. Sağlam (2012) performed comparisons between three mathematics textbooks selected from Turkey, Singapore and the International Baccalaureate Diploma Programme (IBDP) based on their chapters regarding quadratic equations, inequalities and functions. In her study, Sağlam determined that Turkish textbook presented mathematical concepts according to an inductive approach that progressed from equations to functions, while the two other textbooks followed a deductive approach that progressed from functions to equations. Li (2000) described that analyzing how textbooks present subjects along with the structure of the problems they contain will allow more detailed information to be gained during the comparison of textbooks, and that doing so will also contribute to gaining a better perspective of the larger picture. For this reason, the current study did not only analyze how textbooks presented addition and subtraction with integers, but also aimed to provide detailed descriptions regarding the structure of mathematical problems. In parallel to the definitions used in previous studies which compared textbooks (Alajmi, 2012; Li, 2000; Son, 2012; Son & Senk, 2010); this study defined mathematical problems as the mathematical activities, homework and exercises regarding the addition and subtraction of integers which the students were expected to complete. Although researchers do not use the same terms in all studies, mathematical problems are generally analyzed in studies involving the comparison of textbooks by using one or several of the categories described below.

1. *Number of Steps:* Solving the problems requires single or multiple computational procedures (Li, 2000; Son, 2005; Son & Senk, 2010; Stigler, Fuson, Ham & Kim, 1986).
2. *Contextual features:* Involves the analysis of whether the problems are presented in a purely mathematical context, through association with visual representations, or stories (Alajmi, 2012; Li, 2000; Son, 2005; Son & Senk, 2010; Stigler, Fuson, Ham & Kim, 1986).
3. *Response type:* Involves the analysis of whether the solution to the problem requires a numerical response, a numerical expression, or a description (Li, 2000; Son, 2005; Son & Senk, 2010).
4. *Cognitive Requirements:* The types of skill and knowledge required for solving the problem (Li, 2000; Son, 2005, 2012; Son & Senk, 2010).

Li (2000) used the four categories mentioned above to analyze the problems involving addition and subtraction with integers in American and Chinese textbooks. Li determined that the large majority of the problems in the American and Chinese textbooks required single-step solutions; that these problems were mostly presented in a purely mathematical context; and that the American textbook focused more on conceptual understanding. However, Li's study did not describe the approach these textbooks used for teaching addition and subtraction with integers. In addition, the study did not explain the types of models used in the analyzed problems. Son (2005) previously analyzed Korean and American textbooks based on the multiplication and division with fractions. The study included the analysis of problems with fractions, as well as the evaluation of methods used for teaching concepts. Son described that the American textbook placed greater emphasis on presenting algorithms in a conceptual format. He also described that the large majority of the problems in both textbooks did not only focus on procedural understanding, but were also described in a purely mathematical context. Son and Senk (2010) analyzed how multiplication and division with fractions were presented in American and Korean textbooks, and evaluated the types of problems these textbooks included. They determined that the American textbook primarily emphasized conceptual understanding, focusing to a lesser extent on procedural skills; while the Korean textbook simultaneously focused on the development of both of these skills. In addition, it was determined that the large majority of the mathematical problems required single-step numerical solutions, and that they mainly focused on procedural knowledge. Son (2012) analyzed problems involving addition and subtraction with fractions in American and Korean textbooks in terms of their cognitive requirements. The results of the study demonstrated that the American textbooks placed greater emphasis on conceptual knowledge, representations and the solving of verbal problems than the Korean textbooks. Alajmi (2012) performed comparisons between Japanese, Kuwaiti and American elementary school mathematics textbooks with regards to the teaching of fractions. In this study, it was determined that the American textbook contained more pages, and that it included more problems associated with daily life. Alajmi also observed that all of the evaluated textbooks focused on standard definitions and procedural practices.

The current study aimed to demonstrate the similarities and differences between Turkish and American textbooks by analyzing how they presented addition and subtraction with integers, and by evaluating the types of mathematical problems they used.

Method

In this study, data were collected by using the document review technique, included in a qualitative data collection method. According to Yıldırım and Şimşek (2008), document review involves the analysis of written materials which contain information regarding the subject or case being investigated. As this study aimed to compare Turkish and American textbooks in terms of addition and subtraction with integers, the document review technique was applied.

Material Analysis

The study data were obtained through the analysis of the sections regarding the teaching of addition and subtraction with integers in Turkish and American seventh grade textbooks. In Turkey; in case a textbook is deemed to be compatible with the purposes and objectives of the program it is associated with, it will be recommended by the Turkish Board of Education (*Talim Terbiye Kurulu Başkanlığı*, TTKB) for use in mathematics classes. Ubuz, Erbaş, Çetinkaya and Özgeldi (2010) analyzed the characteristics of the mathematics activities within the 6^{th}-8^{th} grade mathematics teaching program, which was first introduced in 2005. These researchers described that the mathematics curriculum guidebook encouraged the development of mathematical reasoning, of complex and non-algorithmic thinking, and of problem-solving skills among the students. In Turkish textbooks that were evaluated and analyzed in this study, the teaching of subtraction and addition with integers is performed at the seventh grade level (MONE, 2009). In this context, we analyzed seventh grade textbooks and workbooks that were prepared in accordance with the 6^{th}-8^{th} grade mathematics teaching program (Sezer et al., 2012; Töker et al., 2012). In addition, as Turkish textbooks are generally published together with their corresponding workbooks; the workbooks were also evaluated and analyzed together with the textbooks. Approval by the TTKB was a criterion in the selection of textbooks for the study. Aside from this, another study criterion was ensuring diversity among the types of textbooks to be analyzed. As it is published by the Turkish Ministry of National Education, the textbook written by Sezer et al. (2012) is able to reach more students. Furthermore, the decision was taken to also include the textbook prepared by Töker et al. (2012), which is published by a private publishing house, into the study analyses. The textbooks prepared by Töker et al. (2012) and Sezer et al. (2012) were coded as TM_1 and TM_2, respectively. TM_1 and TM_2 were approved as textbooks by the TTKB for a period of five years starting from the 2010-2011 and the 2011-2012 academic years, respectively.

The United States does not have a national mathematics curriculum. On the other hand, there are certain mathematics textbooks which are widely used in schools. Within the context of this study, the American textbooks which were evaluated and analyzed included the *MathScape–Course 2* (MS; Educational Development Center [EDC], 2005) and the Connected Mathematics 2 (CM; Lappan, Fey, Fitzgerald, Friel & Phillips, 2009). In addition to addition and subtraction with integers, the MS and CM textbooks also include problems regarding operations with rational numbers, decimal numbers and cartesian coordinates. As Turkish textbooks only included problems regarding addition and subtraction with integers, only these types of problems were considered and analyzed in American textbooks. Similar limitations in the types of problems included were also observed in other studies (e.g., Mayer, Sims & Tajika, 1995*).* MS and CM, in accordance with the principles and standards published by the NCTM (2000), were prepared so as to allow students to develop their own conceptual understanding, to perform mathematical operations, to associate and to make use of mathematical notions (Cai, Moyer, Nie & Wang, 2009; EDC, 2005). Kulm (1999) described that there are only a few elementary school mathematics textbooks in the United States which could be considered as effective, and that MS and CM were among these few that allow students to learn mathematical subjects reliably. Researchers who previously conducted studies on textbooks (Sun, Kulm & Capraro, 2009; Van Garderen**,** Scheuermann & Jackson, 2012) have included and analyzed the MS and CM textbook in their studies by defining them among "reform-based program materials." On the other hand; Li, Chen and An (2009) have described that the MS and CM textbooks belong to the group of widely used textbooks in the United States.

Analysis Regarding the Teaching of Concepts

This analysis aimed to determine and describe the approach used and the teaching models adopted by Turkish and American textbooks in order to develop conceptual understanding and procedural skills among students regarding addition and subtraction of integers. In the analyses performed on this subject, emphasis was placed on determining the sequence followed and the models used to develop conceptual understanding (Alajmi, 2012; Mayer, Sims & Tajika, 1995; Son & Senk, 2010). Mayer, Sims and Tajika (1995) previously described that meaningful teaching methods emphasized the relationships between multiple representations. In this context, the current study did not only analyze whether Turkish and American textbooks utilized verbal, symbolic and visual representations in the teaching of addition and subtraction with integers, but also analyzed how these different representations were associated with one another.

In order to determine the approach used by each one of the textbooks for teaching addition and subtraction with integers, the three researchers separately performed analyses on the textbooks and held notes. Afterwards, the researcher compared their analyses to determine and describe the approach used in the textbooks for teaching addition and subtraction of integers. Based on the obtained data, the similarities and differences between Turkish and American textbooks with regard to the teaching sequence used during the teaching of addition and subtraction with integers were determined.

The Analysis of Mathematical Problems in the Textbooks

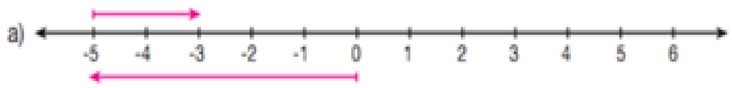
The mathematical problems in the textbooks were analyzed according to their context features and their cognitive requirements. Cognitive requirements are defined as the knowledge and skills which students must acquire while doing mathematics (Son & Senk, 2010). The framework for the analysis used in this study is provided in Table 1.

Table 1. Framework for Problem Analysis

Features	Categories
1. Context features	Purely mathematical context in numerical or word form (PM)
	Illustrative context with pictorial representation or story (IC)
2. Cognitive requirements	
	Procedural practice(PP)
	Conceptual knowledge (CK)
	Representation (R)
	Cubes/stamp/chip models (CSCM)
	Number line model (NLM)
	Verbal problem solving (PS)
	Join (J)
	Separate (S)
	Compare (C)
	Part-Part-Whole (PPW)
	Mathematical reasoning (MR)
	Problem Posing (P. Pos.)

The *contextual features* category provides information on whether problems are presented in a purely mathematical context, as a picture, or through association daily life events. The cognitive requirements of mathematical problems were analyzed according to the following categories: *procedural practice*, *conceptual knowledge*, *representation*, *mathematical reasoning*, *problem solving* and *problem posing*. The *procedural practice* category includes problems which only require the use of operations and algorithms to be solved. The *conceptual knowledge* category includes problems which involve addition and subtraction with integers, as well as questions regarding the underlying meaning of concepts. The *representation* category includes problems with require the utilization of visual representations, the drawing of shapes, or the interpretation of diagrams. Depending on the types of representations used for solving them, problems falling in the representation category are further categorized as cube/stamp/chip models or number line models. The *verbal problem solving* category includes the solving of problems relating to daily life. Problems in this category are further categorized into the *join*, *separate*, *part-part-whole* and *compare* categories (Carpenter, Fennema & Franke, 1996; Son, 2012; Van de Walle, 2007). Problems falling in the *join* category involve processes consisting of start quantity, change quantity, and result quantity; while problems in the *part-part-whole* category involve the combination of two pieces that can be used to create a whole (Van de Walle, 2007). Problems of the *separate* category involve the largest start quantity, from which a certain change quantity is separated; while problems of the *compare* category involve the comparison of two quantities (Van de Walle, 2007). In the *mathematical reasoning* category, problems require that the solution is also described, or that the strategy use is also assessed. The *problem posing* category involves the generation of new problems and the reformulating of existing problems (English, 2003; NCTM, 2000; Silver, 1994). Examples for each one of these categories are provided in Table 2.

Table 2. Sample Textbook Problems and Coding

	Examples	Coding	
		Context	Cognitive Expectation
1.	Write the mathematical sentences which describe the operations modeled on the number line. 	IC	R-NLM MR
2.	For items 9-13, solve each problem by drawing pictures of cubes. 9. 3 + (−1) 10. 5-(-3) 11. −2 + (−3) 12. -4-(-1) 13. −6 − 4	PM	R-CSCM
3.	"I noticed that a negative number minus a positive number will always be negative, because the subtraction makes the answer even more negative." For the statement, decide if the statement will always be true. If the statement is not always true, show an example for which it is false (a counterexample). If the statement is always true, present an argument to convince others that no counterexample can exist.	PM	MR
4.	The apartment building named "Öztürk" has 4 floors above ground and 2 floors below ground. How many floors are there between a family living at floor 2 and a family living at floor –1? Use counting chips to model this operation.	GG	PS-C R-CSCM
5.	Write a story about temperature, Money, or game scores to represent each number sentence. a. +7-(-4)=+11 b. -20+(+n)=+30 c. –n+(-150)--350	PM	P. Pos.
6.	Perform the addition operations provided below. a. (-9)+(-25) b. (-27)-(-19)	PM	PP

* Problems number 1 and 6 were obtained from pages 2 and 3 of the TM_2 textbook; problems 2 and 3 were obtained from pages 124 and 97 of the MS textbook, respectively; problem 4 was obtained from page 15 of textbook TM_1; and problem 5 was obtained from page 34 of textbook CM.

As the first problem in Table 2 was presented visually, it was coded as IC. As the solving of this problem required the use and description of a representation, it was also coded as R-NLM and MR. As in this problem, certain other problems within the textbooks also required the use of more than one skills with respect to cognitive requirements. As the second problem above had a purely mathematical context, it was coded as PM. As the solving of the problem required the use of the cube model, it was coded as CSCM under the R category. Since this problem involved five operations, the coding was performed separately for each operation. As the third problem also had a purely mathematical context, it was coded as PM as well. In this problem, an expression was provided regarding addition and subtraction of integers, and the students were required to describe their thoughts concerning the mathematical correctness of this expression. In addition, the students were also asked to convince their classmates regarding the accuracy of the description they provided. For this reason, this problem was coded as MR. As the fourth problem was presented through associations with daily life, it was coded as IC with respect to context features. On the other hand, with respect to cognitive requirements, the problem was coded as PS. Since the problem asked for a comparison of the distance between two building floors, it was coded within the C sub-category. In the following stage of the problem, the operation used for solving the problem was asked to be modeled with the aid of counting chips. As the solving of the problem required the use of counting chips, it was coded as R-CSCM. As the fifth problem had a purely mathematical context, it was coded as PM with respect to context features. In addition, as this same problem also required for a verbal story to be formed in accordance with the relevant operation, the problem was coded as P.Pos. with respect to cognitive requirement. In the final problem, since the problems had a purely mathematical context, it was coded as PM with respect to context features. However, with respect to cognitive requirements, as the problems required only knowledge of procedural practices, it was coded as PP. This category represents the most basic level, and does not require the use of higher skills or functions other than the necessary operations. As options a and b in this problem were independent of one another, they were considered and analyzed as two separate problems.

The problems in the Turkish and American textbooks were coded separately by two researchers. The coherence between intercoders was calculated using the formula developed by Miles and Huberman (1994) ((Agreement/Agreement + Disagreement) x100). Calculations were performed for each category within the framework for analysis. The researchers then met to discuss their analyses. It was observed that the agreement between the analyses varied between 88% and 98%. Following this, the problems which displayed divergences were discussed and reached a consensus regarding the analyses. The study data were presented comparatively with the aid of frequencies and percentage distributions.

Results

Results Regarding the Teaching of Addition and Subtraction with Integers in Turkish and American Textbooks

In the TM_1 textbook, the teaching of addition and subtraction was performed separately. In TM_1, the teaching of addition with integers began with the representation of upwards and downwards movements with respect to the sea level by using the – and + symbols. In addition to this, the textbook also included an activity in which the sum of the temperature values on different days of the week were calculated and modeled with the use of counting chips (Figure 1). In this activity, the verbal problem was solved through modeling with counting chips; however, verbal descriptions which associated with the model and the problem were not provided within the context of the problem. Following this activity, the students were asked to develop a rule regarding addition with integers. The textbook then provided exercises regarding addition with integers, and asked the students to provide their solutions with counting chips or number lines. A similar approach was followed in the teaching of subtraction. The textbook first provided an activity which requires the modeling of the $(-4) - (-7)$ operation with counting chips. This activity included verbal descriptions regarding the relationship between mathematical sentences and visual representations. On the other hand, the activity was not based on problems associated with daily life. Based on the modeling process, the students were asked to develop a rule regarding subtraction with integers. The textbook then provided exercises which students were required to solve with the aid of counting chips. On the other hand, the textbook did not include activities in which subtraction operations were performed with the aid of a number line.

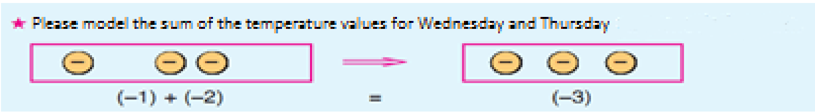


Figure 1. An Example from the TM_1 Textbook Regarding Addition of Negative and Positive Integers (p. 12)

In the TM_2 textbook, the teaching of addition and subtraction was performed separately. The teaching additions with integers began with an activity in which the location of a moving ship was expressed as an integer (Figure 2). On the other hand, this activity did not include verbal explanations or symbolic operations. Following this, the students were asked to model the following question with the aid of counting chips: *"If you are to receive 1 TL, but owe 1 TL, how many liras do you have at hand?"*(Figure 3). This activity did not include verbal descriptions regarding the associations between daily life, symbolic representations and visual representations. Following this, the textbook included activities in which verbal problems relating to daily life were solved with the aid of symbolic and visual representations. Based on these activities, the textbook aimed to discuss the methods which need to be followed for addition operations with integers. In addition to this, the TM_2 also included verbal problems which required the use of the number line for solving. These types of activities also involved the use of verbal explanations.

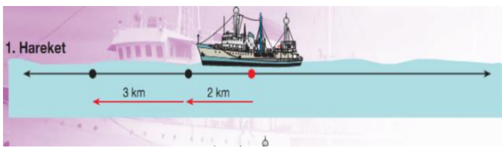


Figure 2. Activity in which the Location of a Ship is Expressed in Integers (p. 2)

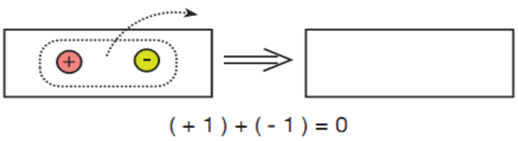


Figure 3. Modeling of the Problem on Receivables and Debts (p. 2)

In TM_2, the teaching of subtraction operations began with an activity in which symbolic operations were modeled with counting chips (*"Let us model the operation* $(+7) - (-4)$ *with counting chips"*, p. 6). This activity did not include verbal problems associated with daily life. On the other hand, the activity did include verbal descriptions regarding the relationship between visual representations and mathematical sentences. Following this activity, the students were asked to make inferences regarding the methods used when performing subtraction operations with integers. The students were then asked to solve the activity by answering the following daily life-related question with the aid of the number line: "*It was observed that the temperature at the foot of the mountain was* -2℃, *while the temperature at the mountain peak was* -11℃. *Based on this information, let us determine the temperature difference between the foot of the mountain and the mountain peak.*"

When teaching the integers, the MS textbook placed emphasis on the understanding of signed numbers and of addition and subtraction. In the section on mathematical reasoning, the textbook focused on the formation and analysis of mathematical expressions regarding addition and subtraction with integers, and also required students to provide reverse examples for mathematical expressions that were incorrect. Using cube models, the MS textbook placed emphasis on activities which allowed students to determine the meaning of addition and subtraction with signed numbers, and on activities which contributed to the development of mathematical reasoning skills of the students. The MS textbook did not include activities which required the use of the number line model. At the beginning of the section on additions and subtractions, students were asked to solve operations such as $6 + (-3)$ and $(-5) - (-4)$ using the cube model. On the other hand, there were no verbal explanations in this section regarding the modeling of mathematical sentences. Following these activities, students were asked to write problems which required the addition of negative or positive cubes, and whose results were known beforehand. The textbook then presented a large number of mathematical expressions involving addition and subtraction with integers, and asked the students to determine whether these expressions always held true. If an expression did not always hold true, the students were required to provide an example of a case in which the expression was incorrect; and if the expression always held true, the students were required to present an explanation on why there were no examples in which the expression was incorrect. An example of an expression used in this activity was as follows: "*I tried four different problems in which I added a negative number and a positive number, and each time the answer was negative. So a positive plus a negative is always negative (p. 97).*" In these types of activities, students are asked to discuss the validity of mathematical expressions, and to present convincing arguments to their peers in case the expressions are valid, or a counterexample in case the expressions are not valid. After these types of activities; another activity was presented in which students were required to form different combinations with a certain number of cubes, and to obtain a specified integer result through addition and subtraction performed with these cubes (e.g., *make all possible combinations of 3 cubes. For each combination, bring in additional cubes so that the overall total is -4; p.98).* The aim of this activity was to perform various addition and subtraction operations using three cubes (the cubes may represent negative or positive integers). For example, the student may write (-2)+(+1) with the aid of three cubes. After writing this operation, the student may add three more negative cubes to obtain a result of –1. Similar activities were repeated throughout the textbook with the aim of ensuring conceptual understanding.

In the CM textbook, the teaching of addition and subtraction was performed separately. The teaching of addition with integers was performed using *part-part-whole* and *joining* category problems regarding addition of natural numbers. The solution for the expression "*John has 8 video games and his friend has 5. Together they have 8+5=13 games*", which belongs to the *part-part-whole* category, was presented by utilizing the set model (Figure 4).

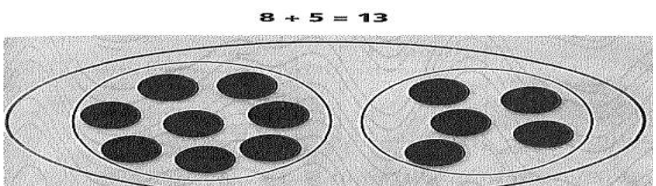


Figure 4. Representation of a Part-Part-Whole Expression with the Set Model (p. 23)

Following this, the CM textbook provided explanations such as "*At a desert weather station, the temperature at sunrise was*10°C. *It rose* 25°C *by noon. The temperature at noon was* 10°C + 25°C = 35°C. *You can represent this situation on a number line. The starting point is +10. The change in distance and direction +25. The sum (+35) is the result of moving that distance and direction*"; these explanations were represented with the number line model, as shown in Figure 5. Following these explanations, students were asked to take into consideration the case when the temperature decreased by 15°C instead of increasing by25°C. The representation of the number line associated with this mathematical sentence is shown in Figure 6. Based on such considerations, the students were asked to develop an algorithm regarding addition operations with integers.

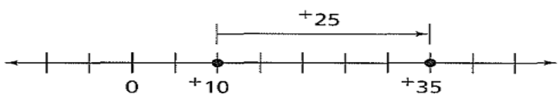


Figure 5. The Numerical Representation of the Expression using the Join Format (p. 23).

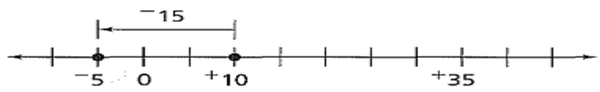


Figure 6. The Modeling with the Number Line Model of the Case where the Temperature drops by 15°C (p. 23).

A similar approach was followed in the CM textbook with regard to the teaching of subtraction operations. For example, the verbal expression "*Kim had 9 CDs. She sold 4 CDs at a yard sale. She now has only 9-4=5 of those CDs left*", which belongs to the *separate* category, was represented using the set model. The expression "*Otis earned $5 babysitting. He owes Latoya $7. He pays her the $5*" was then represented using the chip model, as shown in Figure 7. After providing explanations regarding the modeling process involved in Figure 7, the textbook provided the following problem: "*The Arroyo family just passed mile 25 on the highway. They need to get to the exit at mile 80. How many more miles do they have to drive?*" The problem was solved using the $80 - 25 = 55$ operation, and this solution was further clarified with the number line shown in Figure 8.

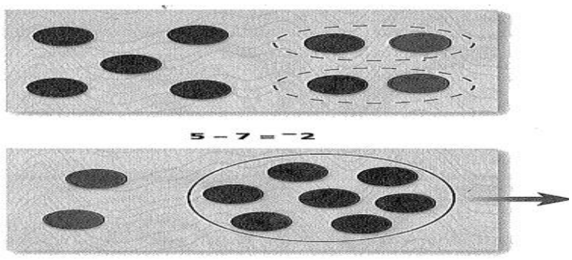


Figure 7. The Representation of the $5 - 7 = -2$ Operation with the aid of the Chip Model (p. 25).

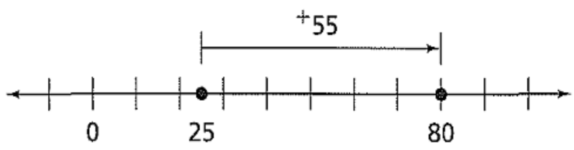


Figure 8. The Modeling of the $80 - 25 = 55$ Operation Using the Number Line (p. 26)

After representing the problem on a number line (as shown in Figure 8), the textbook asked the students to consider the situation in which the car was driven in the opposite direction, from 80 miles to 25 miles. Describing that the car would, in this case, move 55 miles in the opposite direction, the textbook represented the relevant mathematical expression using the number line shown in Figure 9. The textbook indicated that distance traveled in this process was 55 miles, while the direction of travel was opposite, as indicated by the "–" sign. Following these types of activities, the CM textbook included activities regarding the writing of subtraction/addition operations, which were equivalent to addition/subtraction operations. The textbook thus aimed to illustrate the relationship between addition and subtraction (e.g., $+5+?=+2$ and $+5-?=+2$; p. 28). The textbook then provided solutions for various different problems by using the chip and number line models.

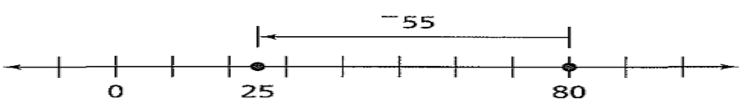


Figure 9. The Modeling with the Number Line of the Distance Traveled in the Opposite Direction (p.26)

Analysis of the Mathematical Problems in Turkish and American Textbooks

The TM_1, TM_2, MS and CM textbooks consisted of 110, 83, 113 and 226 problems. The categorizations according to context feature and cognitive requirements of the mathematical problems regarding addition and subtraction with integers in Turkish and American textbooks are provided in Table 3.

Table 3. Results regarding the Analysis of the Mathematical Problems in Turkish and American Textbooks.

	TM_1 (N=110)	TM_2 (N=83)	MS (N=113)	CM (N=226)
Context features				
Purely mathematical context in numerical or word form	83(75.5)	58(69.9)	110(97.3)	185(81.9)
Illustrative context with pictorial representation or story	27(24.5)	25(30.1)	3(2.7)	41(18.1)
Cognitive requirements				
Procedural practice	61(55.5)	41(49)	39(34.5)	104(46.1)
Conceptual knowledge	2(1.8)	0(0)	0(0)	3(1.1)
Representation				
Cubes/stamp/chip models	28(25.5)	9(10.8)	34(30.1)	18(7.9)
Number line model	6(5.5)	9(10.8)	0(0)	27 (11.9)
Problem solving				
Join	0(0)	6(7.2)	0(0)	6(2.7)
Separate	7(6.4)	0(0)	0(0)	8(3.5)
Compare	4(3.6)	9(10.8)	0(0)	2(0.9)
Part-Part-Whole	0(0)	6(7.2)	0(0)	0(0)
Mathematical reasoning	3(2.7)	13(15.7)	14(12.4)	62(27.4)
Problem Posing	6(5.5)	0(0)	26(23)	22(9.7)

* The data are presented as percentage (frequency).

According to Table 3; in both Turkish and American textbooks, more than 70% of the mathematical problems regarding addition and subtraction with integers were presented in an abstract or verbal format. In addition to this, Turkish textbooks placed greater emphasis on presenting problems regarding addition and subtraction with integers by making associations with visual items or daily life. The TM_2 textbook placed the most emphasis on problems that made associations with visual items or daily life, while the MS textbook placed the least emphasis on such associations.

In the TM_1, TM_2, MS and CM textbooks, the ratio of problems involving addition and subtraction with integers that only required procedural practice to resolve was 55.5%, 49%, 34.5% and 46.1%, respectively. The TM_2 and MS textbooks contained no problems belonging to the conceptual knowledge category, while the ratio of such problems in the TM_1 and CM textbooks was 1.8% and 1.1%, respectively. In the TM_1 and MS textbooks, the ratio of problems which required representation to resolve was above 30%. Although the number of problems which required representation to resolve was greater in the CM textbook; in terms of ratio/proportion, it contained less problems of this of type compared to the other textbooks. Considering the distribution of representations used in the textbooks, it was indicated that the TM_1 and MS textbooks made greater use of the cube, stamp or chip models. The MS textbook used a single type of model for the teaching of addition and subtraction with integers.

The MS textbook included no verbal problems which made associations with daily life, while 7.1% of the problems in the CM textbook were verbal problems. The most common category of verbal problem in the CM textbook was the *separate* type problems (3.5%). On the other hand, the CM textbook contained no *part-part-whole* type problems. Turkish textbooks included a higher proportion of verbal problems. Among the two Turkish textbooks, the highest ratio of verbal problems was found in the TM_2 textbook, with a ratio of 25.2%. When the distribution of verbal problems was taken into consideration, it was observed that the TM_1 and TM_2 textbooks contained more *separate* (6.4%) and *compare* (10.8%) type problems, respectively. On the other hand, the TM_1 textbook did not include any problems from the *join* and *part-part-whole* categories, while the TM_2 textbook did not include any problems from the *separate* category. Several examples of problems from the *separate* and *compare* categories in TM_1 and TM_2 – which represent the most common type of problems these textbooks – are provided below:

> *When Esma looked at the thermometer in the morning, she saw that the temperature value was 11°C. If the temperature decreases by 14°C by the end of the day, what °C value will the thermometer show?* (TM_1 workbook, p. 20).

> *Evaluate and complete the table below by determining the differences between the morning and evening temperatures (the table provided the morning and evening temperature for seven days of the week; e.g. on the table, the morning temperature for Monday was*+3℃*, while the evening temperature was*−4℃*)* (TM_2 workbook, p. 2).

It was observed that 12.39% of the problems in the MS textbook (*e.g., Table 2, problem 3*) and 27.4% of the problems in the CM textbook (*e.g., how can you decide if the sum of two numbers is positive, negative, or zero without actually calculating the sum?* p. 41) required mathematical reasoning to resolve. This ratio was lower in Turkish textbooks. In the TM_1 textbook, only 2.7% of the problems required mathematical reasoning to resolve (*e.g., "In an addition operation performed with integers; –24 is the minuend, while – 5 is the subtrahend. Based on this information, is the result of this operation greater or less than +19? Explain your reasoning." TM_1 Workbook, p. 15*). The same ratio was 15.7% for the TM_2 textbook (*e.g., "Discuss with your friends a method which can be used when performing addition operations with integers." TM_2 Textbook, p. 3*).

The ratio of problems in the MS and CM textbooks that involved problem posing activities was 23% and 9.7%, respectively (*e.g., Table 2, Problem 5*). Among Turkish textbooks; the TM_2 textbook contained no problems that involved problem posing activities regarding addition and subtraction with integers, while 5.5% of the problems in TM_1 textbook required problem posing. An example of a problem posing activity from the TM_1 textbook is provided in Figure 10.

Write a problem using the pictures below. When writing your problem, pay attention to using meaningful and ordered sentences, and on making associations between the problem and daily life. Afterwards, solve the problem you have prepared.

Figure 10. The Problem Posing Activity in Turkish Textbook (TM_1, p. 23).

Discussion, Conclusion and Suggestions

This study aimed to perform a comparison between the reform-based MS and CM textbooks and the TM_1 and TM_2 textbooks prepared according to the 6^{th}-8^{th} Grade Mathematics Program introduced in 2005 with respect to the approaches they used for teaching addition and subtraction with integers, and the types of problems they contained. Thus, it was aimed to contribute to the possible explanations regarding the performance differences observed in comparative international studies.

Several similarities and differences were identified between Turkish and American textbooks with regards to the teaching of addition and subtraction with integers. Except for the MS textbook, three of the evaluated textbooks presented addition and subtraction with integers in two separate sections. In addition, it was observed that Turkish textbooks and the CM textbook placed greater emphasis on the coordination between daily life, visual representations and mathematical sentences, and that these textbooks focused on modeling operations with the aid of cubes/chips/stamps and number lines. On the other hand, the MS textbook used only the cube model for teaching purposes. Furthermore, the TM_2 and CM textbooks provided several examples before developing and presenting rules regarding addition and subtraction of integers. In this respect, the TM_2 and CM textbooks were similar.

Turkish and American textbooks were different from one another with respect to their emphasis on making associations between visual representations, mathematical sentences and examples from daily life. Compared to American textbooks, Turkish textbooks placed less emphasis on verbal expressions that described the association between mathematical sentences, daily life and visual representation. In addition, rather than presenting examples from daily life when teaching subtraction operations, Turkish textbooks largely focused on the relation between direct visual representations and mathematical sentences. While the TM_1 textbook asked students to develop rules regarding operations in question after providing only a single example, the TM_2 textbook asked students to develop rules after providing more than one examples. In this respect, the TM_2 textbook provided more opportunities than the TM_1 textbook for the development of conceptual understanding regarding operation with integers. The CM textbook modeled its teaching of addition and subtraction with integers on different problem structures it previously used when teaching addition and subtraction with natural numbers. Thus, based on the modeling performed with a natural number problem belonging to the *join* category, the CM textbook asked students to perform the modeling of a similarly structured problem the used signed numbers. On the other hand, Turkish textbooks did not include such structures when teaching addition and subtraction with integers. Furthermore, in contrast to Turkish textbooks, the CM textbook made use not only of the cube model at the beginning of its teaching process, but also of the number line model. Unlike Turkish textbooks, the CM textbook established its teaching approach for subtraction operations on examples from daily life. Mayer, Sims and Tajika (1995) described that meaningful teaching methods tend to emphasize the relationships between multiple representations. In this respect, it is possible to state that the CM textbook provided more opportunities for conceptual learning than the other textbooks.

It was determined that 52.8% (102) of the 193 problems in Turkish textbooks and 42.2% (143) of the 339 problems in American textbooks required procedural skills. In their study investigating the teaching of mathematical problem-solving skills in American and Japanese textbooks; Mayer, Sims and Tajika (1995) described that American textbooks provided a higher ratio of exercise type problems (e.g., -8+3=?). The performance level of Japanese students in international comparative studies such as the TIMSS and PISA is higher than that of American students, while the performance level of American students is higher than that of Turkish students. Thus, it is possible to state that student performance is inversely correlated with the number of problems regarding procedural practice that is included in textbooks. This result is similar to the observations made concerning the ratio of problems involving mathematical reasoning and problem posing, which require higher cognitive skills.

In American textbooks, problems involving mathematical reasoning and problem posing represented 22.4% and 14.2% of the total number of problems, respectively. In Turkish textbooks, these ratios were 8.3% and 3.1%, respectively. Thus, one of the reasons why Turkish students demonstrate lower performance than American students in international comparative studies is the low number of problems that require higher cognitive skills in textbooks. Son and Senk (2010) described that problems in Korean textbooks were cognitively more advanced than the problems in American textbooks. The TIMSS and PISA reports indicate that Korean students perform better than American students. When studies from the literature are evaluated and considered together with the results of the current study, it is possible to see that there is a positive relationship between the number of problems requiring higher cognitive skills and the performance level of students. The fact that teachers frequently make use of textbooks for developing classroom activities; the fact that the large majority of teachers are dependent on textbooks (Törnroos, 2001); and the fact that the teaching activities of teachers influence the performance and success of students (Ball, Lubienski & Mewborn, 2001; Ma, 2010) render this relationship between higher cognitive skill problems and student performance even more significant.

Researchers consider problem posing as one of the types of activities that require high levels of cognitive skills (Cai & Hwang, 2002; Dickerson, 1999; Işık & Kar, 2012; Li, 2000). In a study analyzing American and Chinese textbooks, Li (2000) determined that an average of 2% of the problems in American textbooks consisted of problem posing activities. In the current study, this ratio was determined as 14.2% for American textbooks. The MS and CM textbooks have been prepared in accordance with the standards of NCTM (2000) (Cai, Moyer, Nie & Wang, 2009; EDC, 2005). In light of these results, it is possible to observe a development in American textbooks with regard to the inclusion of a large number of problem posing activities. In addition to this, Zhu and Fan (2006) observed that approximately 15% of the problems in American and Chinese textbooks consisted of problem posing activities. In contrast, the ratio of problem posing activities in Turkish textbooks is fairly low. The TM_2 textbook did not include any problem posing activities, while only 5.5% of the problems in the TM_1 textbook consisted of problem posing activities. Similarly, Dede and Yaman (2005) described that 6th, 7th and 8th grade textbooks in Turkey did not include an adequate number of problem posing activities.

It was determined that 26.9% (52) of the 193 of the problems in Turkish textbooks and 23.3% (79) of the 339 problems in American textbooks required representation to be resolved. Based on these results, it is possible to observe that Turkish and American textbooks did not different significantly from one another with respect to the ratio of problems that required representation to be resolved. Compared to the TM_2 textbook, the TM_1 textbook included more problems that required representations. Between the two American textbooks, the MS textbook contained a higher ratio of problems that required the use of representations to be resolved. When the distributions of the types of representations were evaluated, it was determined that the use of the number line and stamp/chip/cube models was not proportionally distributed in the textbooks, and that the stamp/chip/cube models tended to be used more frequently. Compared to the other textbooks, the TM_2 and CM textbooks showed a relatively more proportional distribution of problems that required the use of both models.

The TM_2 textbook included more verbal problems than the TM_1 textbook. Between the two American textbooks, the MS textbook did not contain any verbal problems, while 7.1% of the problems in the CM textbook were verbal problems. The average ratios of verbal problems in Turkish and American textbooks were 16.6% and 4.8%, respectively. In light of these results, it is possible to state that Turkish textbooks placed more emphasis on making associations with examples from daily life situations. American and Turkish textbooks did not include the whole range of problem types – which comprised problems from the *join*, *separate*, *compare* and *part-part-whole* categories. The TM_1 textbook did not include problems from the *join* and *part-part-whole* categories, while the TM_2 textbooks did not include problems from the *separate* category. On the other hand, compared to the other textbooks, the TM_2 textbook showed a more proportional distribution with respect to its verbal problems. The meaning of an operation is shaped by the contextual structure of the relevant problem (i.e., the types of actions or relations described in the problem) (Carpenter, Fennema & Franke, 1996). The NCTM (2000) emphasizes that students should know and recognize all types of problems structures, since such knowledge would assist them in conceptualizing the different meanings of different operations associated with these problems. On the other hand, it is possible to note that both Turkish and American textbooks did not include problems which reflected the different meanings of addition and subtraction with integers.

In this study, it was determined that the CM textbook presented more opportunities for the effective teaching of addition and subtraction with integers. In addition, it was also determined that American textbooks provided more problems that required higher cognitive skills to resolve, such as problems based on mathematical reasoning and problem posing. On the other hand, it was observed that Turkish textbooks included more problems associated with procedural skills. These findings and results contribute to the explanation of the reasons for the lower performance observed with Turkish students in PISA and TIMSS reports in comparison to American students. Furthermore, the results of the current study were in parallel with the findings of other studies which analyzed Turkish textbooks. For example, İskenderoğlu and Baki (2011) previously analyzed a Turkish eighth grade mathematics textbook according to the PISA mathematics competency scale. The mathematics competency scale has six levels. İskenderoğlu and Baki determined that Turkish textbooks do not include fifth and sixth level problems, and that the large majority of their problems were second level problems. This second level corresponds to procedural skills. With the revised 5^{th}-8^{th} Grade Mathematics Program (MONE, 2013), efforts are being made to reorganize textbooks in Turkey. This study presents important information and results regarding the approaches used in textbooks for the teaching of addition and subtraction with integers, and the structure of the problems used in these textbooks. Thus, these results provide textbook authors the opportunity to view the preparation of textbooks from a broader perspective. In addition, this study will provide teachers with information on how American textbooks teach addition and subtraction with integers, and hence contribute to the organization of the teaching activities conducted by Turkish teachers. In this study, the analysis of textbooks was limited to addition and subtraction with integers. However, in order to gain a broader understanding regarding textbook, it is necessary to conduct studies on mathematical subjects other than addition and subtraction with integers. This study focused on the contextual and cognitive characteristics of problems in textbooks. As such, the study analyzed the width of knowledge textbooks provided rather than the depth of mathematical knowledge being provided in them. In addition, by analyzing the structure of problems with the aid of the analysis frameworks described in the literature (Son, 2012; Webb, 2002) for the evaluation of cognitive depth in mathematical knowledge; it will be possible to assist textbook authors during the preparation stages of new textbooks. This study was limited to the analysis of only two Turkish and two American textbooks. For this reason, conducting analyses on a larger number of textbooks would further contribute to understanding the causes underlying the performance difference observed in international comparative studies between Turkish and American students.

References

Aktaş, M. C., & Aktaş, D. Y. (2012). İlköğretim 7. sınıf matematik öğretim programı, ders ve öğrenci alışma kitaplarında dörtgenler arasındaki ilişkilerin anlatımının incelenmesi. *E-Journal of New World Sciences Academy, 7*(2), 848-858.

Alajmi, A. H. (2012). How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics*, *79*(2), 239-261.

Altınok. A., Keşan, C., & Yılmaz, S. (2005). İlköğretim 7. Sınıf tamsayılar konusunun günlük yaşamla ilişkilendirilmesi ve öğrenci üzerindeki etkisi. *XIV. Ulusal Eğitim Bilimleri Kongresi,* Denizli.

Altun, M., Arslan, Ç., & Yazgan, Y. (2004). Lise matematik ders kitaplarının kullanım şekli ve sıklığı üzerine bir çalışma. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 17*(2), 131-147.

Arslan, S., & Özpınar, İ. (2009). İlköğretim 6. Sınıf matematik ders kitaplarının öğretmen görüşleri doğrultusunda değerlendirilmesi. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 12,* 97-113.

Avcu, T., & Durmaz, B. (2011). Tam sayılarla ilgili işlemlerde ilköğretim düzeyinde yapılan hatalar ve karşılaşılan zorluklar. *2nd International Conference on New Trends in Education and Their Implications,* Antalya-Turkey.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. *Handbook of Research on Teaching*, *4*, 433-456.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21,* 401-421.

Cai, J., Moyer, J., Nie, B. & Wang, N. (2009). Learning mathematics from classroom instruction using Standards-based and traditional curricula: An analysis of instructional tasks. In S. L. Swars, D. W. Stinson & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 692-699). Atlanta: Georgia State University.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, *97*(1), 3-20.

Dede, Y., & Yaman, S. (2005). İlköğretim 6. 7. ve 8. Sınıf matematik ve fen bilgisi ders kitaplarının incelenmesi: Problem çözme ve problem kurma etkinlikleri bakımından. *XIV. Ulusal Eğitim Bilimleri Kongresi'nde sunulmuş bildiri, Denizli.*

Dickerson, V. M. (1999). *The impact of problem-posing instruction on the mathematical problem solving achievement of seventh graders (Unpublished doctoral dissertation).* University of Emory, Atlanta.

Education Development Center [EDC], (2005). *MathScape: Seeing and thinking mathematically: Course 2.* Columbus, OH: Glencoe/ McGraw-Hill.

English, L. D. (2003). Problem posing in elementary curriculum. In F. Lester, & R. Charles (Eds.), *Teaching mathematics through problem solving.* Reston, Virginia: National Council of Teachers of Mathematics.

Erbaş, A. K., Alacacı, C., & Bulut, M. (2012). Türk, Singapur ve Amerikan matematik ders kitaplarının bir karşılaştırması. *Kuram ve Uygulamada Eğitim Bilimleri, 12*(3), 2311-2330.

Fan, L., & Zhu, Y. (2000). Problem solving in Singaporean secondary mathematics textbooks. *The Mathematics Educator, 5*(1/2), 117-141.

Gallardo, A. (2002). The extension of the natural-number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics, 49*(2), 171-192.

Işık, C., & Kar, T. (2012). İlköğretim matematik öğretmeni adaylarının kesirlerde bölmeye yönelik kurdukları problemlerde hata analizi. *Kuram ve Uygulamada Eğitim Bilimleri 12*(3), 2289-2309.

İskenderoğlu, T. A., & Baki, A. (2011). İlköğretim 8. Sınıf matematik ders kitabındaki soruların PISA matematik yeterlik düzeylerine göre sınıflandırılması. *Eğitim ve Bilim, 36*(161), 287-301.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kulm, G. (1999). Evaluating mathematics textbooks. *Basic Education*, *43*(9). http://www.project2061.org/publications/articles/articles/cbe.htm adresinden 12 Haziran 2013 tarihinde alınmıştır.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Connected Mathematics 2, Grade Seven.* Pearson Prentice Hall.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education*, 234-241.

Li, Y., Chen, X., & An, S. (2009). Conceptualizing and organizing content for teaching and learning in selected Chinese, Japanese and US mathematics textbooks: The case of fraction division. *ZDM*, *41*(6), 809-826.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Newyork, NJ: Lawrence Erlbaum Associates.

Mayer, R. E., Sims, V., & Tajika, H. (1995). A comparison of how textbooks teach mathematical problem solving in Japan and the United States. *American Educational Research Journal*, *32*(2), 443-460.

Mesa, V. (2004). Characterizing practices associated with functions in middle school textbooks: An empirical approach. *Educational Studies in Mathematics, 56*, 255-286.

Miles, M. B., & Huberman, M. A. (1994). *An expanded sourcebook qualitative data analysis.* London: Sage.

Ministry of National Education [MONE], (2009). *Elementary mathematics curriculum and manual: Grades 1-5,* Ankara.

Ministry of National Education [MONE], (2013). *Middle School mathematics curriculum: Grades 5-8,* Ankara.

Milli Eğitim Bakanlığı, Eğitim Araştırma ve Geliştirme Dairesi Başkanlığı [MEB-EARGED] (2003). TIMMS 1999: Üçüncü uluslar arası matematik ve fen bilgisi çalışması ulusal rapor. http://earged.meb.gov.tr/dosyalar/dokumanlar/uluslararasi/timms_1999_ulusal_raporu.pdf adresinden 27 Mayıs 2013 tarihinde alınmıştır.

National Council of Teachers of Mathematics [NCTM], (2000). *Principles and standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

Organisation for Economic Cooperation and Development [OECD], (2006). Database-PISA 2006. http://www.oecd.org/pisa/pisaproducts/pisa2006/pisa2006results.htm adresinden 25 Mayıs 2013 tarihinde alınmıştır.

Organisation for Economic Cooperation and Development [OECD], (2009). Database-PISA 2009. http://www.oecd.org/pisa/pisa2009keyfindings.htm adresinden 25 Mayıs 2013 tarihinde alınmıştır.

Sağlam, R. (2012). *Türkiye, Singapur ve Uluslararası Bakalorya Diploma Programı'nın matematik ders kitaplarında ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar konusunun karşılaştırmalı bir analizi (Yayınlanmamış yüksek lisans tezi).* Bilkent Üniversitesi, Ankara.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D.E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dorddrecht: Kluwer Academic.

Sezer, R., Demirel, T., Arıcı, Ö., Boztaş, N., Yıldırım, O., Aksoy, Ö. U., & Dumlupınar, M. A. (2012). *İlköğretim matematik 7 ders kitabı.* Devlet Kitapları (Birinci Baskı), Ankara.

Sezer, R., Demirel, T., Arıcı, Ö., Boztaş, N., Yıldırım, O., Aksoy, Ö. U., & Dumlupınar, M. A. (2012). *İlköğretim matematik 7 öğrenci çalışma kitabı.* Devlet Kitapları (Birinci Baskı), Ankara.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Son, W. J. (2005). A Comparison of how textbooks teach multiplication of fractions and division of fractions in Korea and in the U.S. In: H. L. Chick and J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education,* 4, 201-208.

Son, J. W. (2012). A cross-national comparison of reform curricula in Korea and the US in terms of cognitive complexity: the case of fraction addition and subtraction. *ZDM, 44,* 161-174.

Son, J. W., & Senk, S. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74*(2), 117-142.

Sood, S., & Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *The Journal of Special Education, 41*(3), 145-157.

Steiner, C. J. (2009). *A study of pre-servıce elementary teachers' conceptual understanding of integers (unpublished doctoral dissertation). Kent State University College and Graduate School of Education, US.*

Stigler, J. W., Fuson, K. C., Ham, M., & Kim, M. S. (1986). An analysis of addition and subtraction word problems in American and Soviet elementary mathematics textbooks. *Cognition and Instruction, 3,* 153-171.

Sun, Y., Kulm, G., & Capraro, M. M. (2009). Middle grade teachers' use of textbooks and their classroom instruction. *Journal of Mathematics Education*, *2*(2), 20-37.

Töker, Z., Laçin, O., Kasal, N., Çetinkaya, G., Kebapçı, İ., Akyıldız, M., & Ahioğlu, N. (2012). *İlköğretim matematik ders kitabı 7.* Semih Ofset Matbaacılık, Ankara.

Töker, Z., Laçin, O., Kasal, N., Çetinkaya, G., Kebapçı, İ., Akyıldız, M., & Ahioğlu, N. (2012). *İlköğretim matematik öğrenci çalışma kitabı 7.* Semih Ofset Matbaacılık, Ankara.

Törnroos, J. (2001). Finnish mathematics textbooks in grades 5-7. *Paper presented at the 2nd Scandinavian Symposium on Research Methods in Science and Mathematics Education*, Helsinki.

Trends in International Mathematics and Science Study [TIMSS], (2007). Database-TIMSS 2007. http://timss.bc.edu/timss2007/mathreport.html adresinden 25 Mayıs 2013 tarihinde alınmıştır.

Tutak, T., & Güder, Y. (2012). İlköğretim 5. sınıf öğretmenlerinin matematik ders kitabı hakkındaki görüş ve düşünceleri. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 19,* 16-28.

Türnüklü, E. B., & Yeşildere, S. (2007). The pedagogical content knowledge in mathematics: PRE-servıce primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1,* 1-13.

Tyson, H., & Woodward, A. (1989). Why students aren't learning very much from textbooks. *Educational Leadership, 47*(3), 14-17.

Ubuz, B., Erbaş, A. K., Çetinkaya, B., & Özgeldi, M. (2010). Exploring the quality of the mathematical tasks in the new Turkish elementary school mathematics curriculum guidebook: the case of algebra. *ZDM*, *42*(5), 483-491.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht: Kluwer.

Van de Walle, J. A. (2007). *Elementary and middle school math-ematics: Teaching developmentally* (6th ed.). Boston, MA: Pearson Education

Van Garderen, D., Scheuermann, A., & Jackson, C. (2012). Developing representational ability in mathematics for students with learning disabilities a content analysis of grades 6 and 7 textbooks. *Learning Disability Quarterly*, *35*(1), 24-38.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in `negativity'. *Learning and Instruction, 14*(5), 469-484.

Webb, N. L. (2002). An analysis of the alignment between mathe-matics standards and assessments for three states. *Paper presented at the annual meeting of the American Educational Research Association*, New Orleans, LA.

Yıldırım, A., & Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri*. Seçkin yayıncılık (6. Baskı), Ankara.

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and The United States. *International Journal of Science and Mathematics Education*, *4*(4), 609-626.