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Learning Geometric Translations in a Dynamic Geometry Environment

Öteleme Dönüşümünün Dinamik Geometri Ortamında Öğrenimi

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Abstract

Dynamic learning environments provide a variety of opportunities for learners to explore mathematical concepts. The purpose of this study was to explore the nature of the growth of 4 prospective middle school mathematics teachers' understanding of geometric translations in a technology mediated environment using GeoGebra as a pedagogical medium. Individual teaching experiment methods were used to examine the progress of prospective teachers' understanding of geometric translations. The study design included three phases: (1) semi-structured clinical interviews; (2) teaching episodes; and (3) a retrospective analysis of the semi-structured clinical interviews and teaching episodes. The findings of the study indicated that the availability of the dynamic geometry software supported the teacher candidates' understanding of geometric translations. Specifically, the dragging and measurement features of the program enabled teacher candidates to explore the properties of geometric translations, make conjectures, employ various strategies, and construct new understandings.

Keywords: Geometric translations, technology, GeoGebra, prospective teachers, teacher education

Öz

Dinamik öğrenme ortamları, matematiksel kavramların keşfedilmesi ve incelenmesi açısından öğrencilere birçok fırsatlar tanımaktadır. Bu çalışmanın amacı, GeoGebra dinamik programının eğitimsel bir araç olarak kullanıldığı bir ortamda 4 ilköğretim matematik öğretmen adayının öteleme kavramının gelişiminin incelenmesidir. Bireysel öğretim deneyi metodu kullanılarak öğretmen adaylarının öteleme dönüşümü üzerine gelişimleri incelenmiştir. Çalışma üç bölümden oluşmuştur: (1) yarı-yapılandırılmış klinik görüşmeler; (2) öğretim deneyleri ve (3) geriye dönük yarı-yapılandırılmış klinik görüşmelerin ve öğretim deneylerinin analizi. Çalışmanın bulguları, dinamik geometri programının öğretmen adaylarının öteleme kavramının gelişimini desteklediğini göstermiştir. Özellikle, programın sürükleme ve ölçme özelliklerinin öğretmen adaylarının öteleme kavramının özelliklerini keşfetmelerine, matematiksel varsayımlarda bulunmalarına, çeşitli yöntemler kullanmalarına ve yeni bilgiler oluşturmalarına yardımcı olduğu gözlemlenmiştir.

Anahtar Sözcükler: Öteleme Dönüşümü, Teknoloji, GeoGebra, Öğretmen Adayları, Öğretmen Eğitimi.

Introduction

Technology is an important tool for learning various content areas including mathematics (Kazu & Yavuzalp, 2008). The appropriate use of technology in teaching and learning mathematical concepts has been considered a crucial factor for high quality mathematics education. According

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to The National Council of Teachers of Mathematics (NCTM) (2000) technology should be integrated in all mathematics subjects at all levels. ''Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning'' (NCTM, 2000, p. 11). Geometry is one of the crucial content areas of mathematics curriculum in which technological tools have been extensively developed and implemented. Among these tools, dynamic programs (such as Geometer's Sketchpad, Cabri, Geometric Inventor and GeoGebra) play a significant role in geometry teaching and learning. Previous studies (e.g., Falcade, Laborde, & Mariotti, 2007; Gawlick, 2002; 2005; Harper, 2003; Hollebrands, 2003; 2007; Laborde, 2001; Ruthven, Hennessy, & Deaney, 2008; Yanik, 2009) suggest that dynamic programs can be used to enhance student learning.

Transformational geometry (translations, reflections, and rotations) is an important area of geometry that provides a range of opportunities for learners to explore variety of concepts. "Mathematical properties from the various branches of geometry (topology, projective geometry, affine geometry, Euclidean geometry) can be described in terms of transformations which may be represented through several types of manipulative activities" (Williford, 1972, p. 260). Dynamic geometry environments provide learners fruitful learning experiences, such as discovering patterns and basic features of isometries, making conjectures and testing ideas, and exploring mathematical relationships (e.g., Glass, 2001; Harper, 2003; Hollebrands, 2007; Yanik, 2009). While studies advocate the use of dynamic geometry programs in teaching and learning of geometric transformations, very little is known about learners' concept formation through the use of technology (Hollebrands, 2003; 2007, Mehdiyev, 2009; Yanik, 2009).

This study was a part of a larger study that investigated the nature of pre-service middle school mathematics teachers' understanding of geometric transformations (translations, reflections and rotations) in the context of a technological environment. This study attempts to discover the growth of prospective middle school mathematics teachers' understanding of geometric translations using GeoGebra dynamic software. For the purpose of the study, the following research question was examined: "How do pre-service teachers' initial understandings of geometric translations evolve during and after instruction using GeoGebra software?"

Understanding Rigid Geometric Translations

This study focused on rigid geometric translations that preserve the relative angles and distances of all points in the plane. Geometric translations can be conceived in two distinct ways: motion and mapping (Edwards, 2003; Hollebrands, 2003; Yanik, 2009, 2011). In the motion conception of translations, all points in the plane can be translated onto new points on the plane based on two parameters (direction and magnitude) generally defined by a translation vector. In this conception, one may conceive the plane as a flat background and manipulate geometric figures on top of the plane (Edwards, 2003; Hollebrands, 2003; Yanik, 2009, 2011). On the other hand, in mapping understanding, motion is not an essential part of transformations. Translations are considered special functions that map all points in the plane to other points in the plane based on a specific direction and magnitude. All points of the plane are conceived as the domain of the function and one needs to apply translations to all points in the plane rather than to individual points or a figure.

Prospective teachers' understanding of geometric translations

Past research indicated that prospective teachers had various difficulties in understanding geometric transformations. Specifically, teacher candidates had difficulties (a) describing geometric translations and vector coherently (Ada & Kurtulus, 2010; Desmond, 1997; Harper, 2003; Jung, 2002; Yanik, 2009, 2011) (b) representing translations using mathematical symbols (Jung, 2002; Yanik, 2009, 2011), (c) finding translation vectors that map pre-image points to image points (Desmond, 1997; Harper, 2003; Yanik, 2009, 2011), (d) identifying and executing translations correctly (e.g., Desmond, 1997; Harper, 2003; Yanik, 2009, 2011), and (e) using formal vocabulary to describe translations (Harper, 2003; Jung, 2002; Portnoy, Grundmeier & Graham, 2006; Thaqi, Gimenez & Rosich, 2011; Yanik, 2009, 2011).

While prior studies revealed prospective teachers' difficulties, only a few of the studies researched the effect of technology on their knowledge and understanding of geometric translations. For instance, Jung (2002) explored two pre-service secondary school mathematics teachers' understanding of transformation geometry (e.g., translation, reflection, glide reflection, and rotation) in a technology-based collegiate mathematics classroom. The results indicated that initially prospective teachers used informal mathematical terminology (e.g., moving, flipping) to describe transformations. The findings of the study further showed that participants mainly used pictorial and verbal descriptions rather than symbolic representations to describe transformations. Working with dynamic geometry program, such as the Geometer's Sketchpad helped the participants make conjectures regarding the features of geometric transformations and provided opportunities to verify them. At the end of the study prospective teachers were able to provide more accurate and clear descriptions about transformations using pictorial, verbal, and symbolic representations.

In her study, Harper (2003) investigated 4 pre-service elementary teachers' understanding of geometric transformations in a technology mediated environment. She found that prospective teachers had difficulty executing and identifying translations accurately. Specifically, using vectors to execute translations and identifying the translation vector for a given pre-image and image points were the major difficulties of prospective teachers. Harper found that using dynamic geometry software, such as The Geometer's Sketchpad can enhance prospective teachers' understanding of geometric translations. In particular Harper stated that "The Geometer's Sketchpad's immediate visual feedback aided the participants to conjecture, test and revise their solutions" (p. 6).

Yanik (2009) investigated a prospective teachers' growth of understanding of geometric translations in a computer- based environment in which The Geometer's Sketchpad was used as a pedagogical tool. The findings of the study indicated that the prospective teacher initially conceived translations as undefined motion and had difficulty describing and performing translations. The dynamic feature of the program such as dragging helped the prospective teacher explore the characteristics of geometric translations. At the end of the study, the participant began partly to see transformations as mappings of the plane onto itself.

While these studies found that dynamic geometry environments promote learning of geometric transformations, very little is known about how learning grows in such an environment. Calder, Brown, Hanley and Darby (2006) acknowledged the importance of pedagogical media on learning mathematical concepts. According to Miera (1995), learners' understandings of a mathematical concept are shaped by their use of technological tools. The purpose of this study was to explore teacher candidates' growing ideas of geometric translations in the context of a technological environment; the dynamic mathematics program, GeoGebra, was utilized.

GeoGebra as a pedagogical medium

The pedagogical media (e.g., internet, spreadsheets, graphical calculators, dynamic geometry software, and computer algebra systems) offers new ways of teaching and learning mathematics (Calder, Brown, Hanley, & Darby, 2006). GeoGebra is one of the dynamic mathematics software and enables learners to explore a variety of mathematical concepts in a dynamic environment. According to Haddas and Hershkowitz (1998), "A main pedagogical feature of many dynamic geometry based learning environments is that the discovery and conviction of geometrical facts is greatly enhanced by means of dynamic processes" (p. 25). In a dynamic geometry environment, one can construct drawings on the computer screen that can be manipulated through dragging and that can represent an entire class of objects rather than a single example (Flanagan, 2001; Ruthven, Hennessy, & Deaney, 2008). "This is different from the static diagram about which students are expected to reason as a general case, because the dynamic drawing carries with it certain behaviors that are related to the properties the construction embodies" (Flanagan, 2001, p. 6).

Methodology

Setting and participants

This study was conducted in a middle school mathematics teacher education program at a large urban public university in Turkey. The study was a follow up study of another study (Yanik, 2011) that investigated 44 second-year prospective middle school mathematics teachers' preconceptions of geometric translations through semi-structured clinical interviews in a non-technological environment. A total of 4 teacher candidates (Selcan, Samet, Reyhan, and Kiraz- all participants are given pseudonyms) in the age range of 19-20 were selected from the previous study to explore their growth in understanding of geometric translations in a technology-mediated environment. Two criteria were used for the selection of the participants: 1) willingness to participate in the current study and 2) having a range of understanding of geometric translations. All participants were very reflective persons, who were ready to accept challenges and were able to share their thoughts explicitly. Furthermore, the previous study (Yanik, 2011) had shown that the four participants held various conceptions of geometric translations (e.g., conceiving translations as translational motion or rotational motion). Both criteria were considered as crucial for the study in terms of determining the possible effects of using technology on participants' growth in understanding of geometric translations. In the teacher education program, geometric translations are introduced to the teacher candidates as part of an Analytic Geometry course in the third year of the program. All participants had taken one introductory Geometry course and three elementary Mathematics courses at the undergraduate level before the study began. The participants stated that they could not remember whether or not they took a geometry course which included geometric translations. However, it was assumed that participants still might have some initial conceptions about translations based on their everyday experiences, previous courses, and common sense. None of the participants had experience using a dynamic geometry program before. Therefore, all participants took 1 hour tutoring sessions on GeoGebra to learn the basics of the program. Additional help provided by the researcher during teaching episodes when the participants needed particular assistance regarding the use of GeoGebra.

Procedure

Individual teaching experiment methods (Steffe & Thompson, 2000) were used to examine the progress of prospective teachers' understanding of geometric translations. The study design included three phases: (1) semi-structured clinical interviews; (2) teaching episodes; and (3) a retrospective analysis of the semi-structured clinical interviews and teaching episodes.

Semi-structured task-based clinical interviews. A semi-structured task-based clinical interview was conducted with each of the participants prior to and following the teaching episodes to investigate teacher candidates' pre-existing and progressing knowledge and understanding of rigid geometric transformations. The initial clinical interviews consisted of 13 tasks regarding geometric translations; GeoGebra was not used.

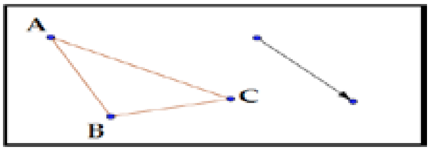
Several resources (e.g., Edwards, 2003; Flanagan, 2001; Molina, 1990; Yanik, 2009) were used to design the interview tasks (see Table 1 for sample tasks). These resources guided the researcher to determine the type of questions (e.g., description tasks, recognition tasks, and performance tasks) asked in the interviews, the order of tasks, and to analyze the teacher candidates' initial understandings of geometric translations.

The interview tasks were designed to understand to what extent teacher candidates could describe, recognize, represent, and perform translations. Furthermore, the interview tasks aimed to understand participants' thinking about the role of vector in translations, and the relationship between the plane and the geometric shapes.

The final clinical interviews aimed at understanding teacher candidates' current constructions of geometric translations. During the final interview, the participants had access to GeoGebra as well as paper and pencil. The interview tasks required participants to reflect on what they had experienced during teaching episodes and allowed the researcher to follow teacher candidates' growth in understanding of geometric translations. The interviews lasted about 60 - 75 minutes. All clinical interviews were video-taped and later transcribed for data analysis purposes.

Table 1.

Sample Interview Tasks

Task type	Relevant tasks
Description task	1) How would you describe a geometric translation? 2) What would be an example of a translation? 3) What would be a non-example of a translation?
Recognition task	4) Can this be an example of a translation? If yes, Why? What would be the translation vector?; If no, Why not?
Performance task	5) Can you perform the following translation? 

Teaching episodes. The researcher carried out the teaching episodes on an individual, one-to-one basis to explore participants' nature of understandings of geometric translations and provide detailed descriptions of their progress. Each teaching episode was designed based on the participants' levels of understanding determined by the initial clinical interviews. For four weeks, the researcher met with each of the participants one at a time once or twice a week for 60-75 minutes for one-on-one instructional sessions in a math lab where participants had access to a laptop and GeoGebra.

The researcher served as the instructor, and engaged in interaction with each participant regarding his/her ideas and explanations about geometric translations. All teaching episodes were video-taped for data analysis purposes. During the first teaching episode the researcher provided several tasks that required participants to execute translations of figures. Participants examined figures and their images under translations and reflected on the results of translations. Specifically, participants were asked to identify features that changed or remained the same. The second teaching experiment focused on the *effect* of multiple translations. Participants were provided multiple vectors and asked to perform translations using the given vectors. The third teaching experiment focused on the domain for translations. The researcher shared the knowledge regarding a translation applies all points in the plane with the participants. Participants were then provided multiple geometric figures to execute translations. During the last teaching experiment, participants were directed to focus on the effects of translations on the locations of the pre-images through the use of coordinate system.

Data analyses. Ongoing and retrospective analyses were conducted throughout the study. While ongoing analysis took place during and between teaching episodes, retrospective analysis occurred at the end of teaching experiment. The purpose of the ongoing analysis was to find patterns and provide detailed descriptions of participants' growth in understanding geometric translations. After each teaching episode the researcher analyzed and coded each participant's responses to the given tasks (see Table 2 for sample codes). To ensure the validity of the codes, three external colleagues also independently critiqued the researcher's decisions and provided feedback regarding the participants' understanding of geometric translations throughout the study. These critiques and feedbacks helped the researcher make more accurate coding decisions and categorization of participants' understanding of geometric translations. Descriptions and hypotheses regarding individual participants' understandings also served as a basis to design subsequent teaching episodes and formulate hypothetical learning trajectories (Simon, 1995) for each participant. The researcher also noted each participant's similarities and differences in case of emerging learning trajectories.

Retrospective analyses took place after the completion of the teaching experiment and involved cumulative analyses of video records of the participants' work. Two rounds of retrospective analyses were conducted. Initially, the researcher reexamined the data gathered through clinical interviews and the teaching episodes to support or challenge the assertions made regarding each participant's understanding of geometric translations throughout the study. The second round sought to find common patterns in participants' interpretations of geometric translations, to investigate the impact of GeoGebra, and examine whether or not the overall hypotheses developed throughout the study were proper.

Table 2.

Sample Codes Generated from the Data

Codes for conceptions of translations and vectors		Sample participant response
T1	Translation as translational motion	"Moving or pushing a geometric shape away (without rotating it."
T2	Translation as rotational motion	"Like rotating a disc without sliding it."
T3	Translation as both translational and rotational motion	"Translation could be achieved through rotational or translational motion."
V1	Vector as a force	"You use vector to push or drag the figure"
V2	Vector as a line of symmetry	"I just took the vector as a symmetry line and performed the translation."
V3	Vector as a direction indicator	"The the vector shows the direction ."

Results

This section, first, describes the prospective teachers' nature of understandings of geometric translations and vector concepts before they interacted with GeoGebra. Then, overall participants' growths in understanding of geometric translations in the dynamic geometry environment are explained. Lastly, the findings of the study are discussed.

Initial Understandings of Geometric Translations

The findings of the study revealed that initially all participants had motion conceptions of geometric translations at various levels of understanding. Three major understandings in relation to translation became apparent as a result of the analyses of the first semi-structured clinical interviews: (a) translation as translational motion, (b) translation as rotational motion, and (c) translation as both translational and rotational motion (see Table 3).

Table 3.

Prospective Teachers' Conceptions of Translations and Vector

	Vector as a symmetry line	Vector as a force	Vector as an abstract tool
Translation as translational motion			Samet
Translation as rotational motion	Kiraz	Reyhan	
Translation as both translational and rotational motion	Selcan		

Translation as translational motion. Samet conceived translation as translational motion. When he was asked to describe a geometric translation, he said: "It's like moving or pushing a geometric shape away." Samet considered the translational motion and the displacement as the main characteristics of a translation. He indicated that if there is no motion, there would be no translation. Samet also considered the plane as a background where geometric shapes could be manipulated. He thought that translation would apply on a single geometric shape rather than all points in the plane. Samet also knew that translation would preserve the shape, size, and the direction of the figure. However in his descriptions for geometric translations, he did not provide any indicator (direction or magnitude) regarding what defines a translation. In this respect, his conception of translation can also be characterized as undefined translational motion.

Translation as rotational motion. Reyhan and Kiraz were the two participants who held the conception of translation as rotational motion. Both participants considered that rolling motion of a single geometric shape and the displacement would be the major characteristics of geometric translations. According to these participants, translation would apply on a single geometric shape and preserve the shape and the size of the figure being translated. Reyhan described a translation as the rolling motion of a disc on an inclined surface. Kiraz gave an example of a rolling wheel on a flat surface to represent a translation (see Figure 1). Both participants considered the plane as a background where geometric figures could be manipulated.

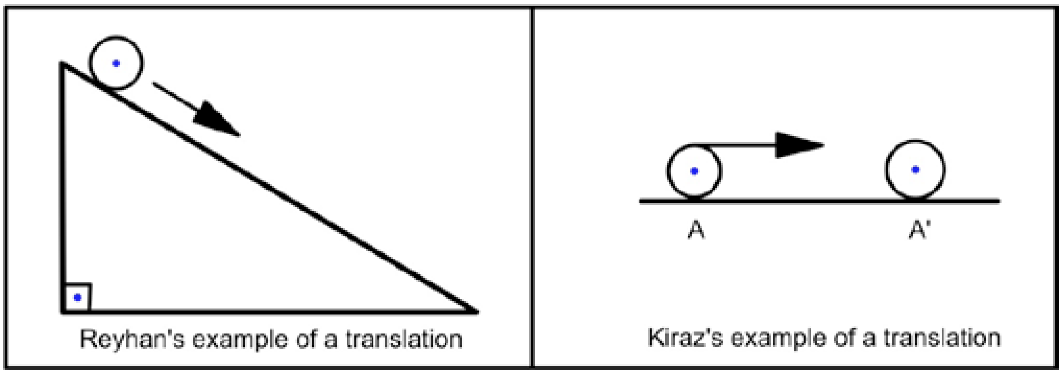


Figure 1. Reyhan's and Kiraz's examples of a translation

Translation as both translational and rotational motion. Selcan also held the motion conception of translation and thought that a translation could be achieved through a translational or rotational motion. For Selcan, the displacement and the motion seemed to be the major features of a translation although she stated that she did not know much about geometric translations.

Initial Understandings of Translation Vector

Three major conceptions regarding translation vector became apparent as a result of the analyses of the first semi-structured clinical interviews: (a) vector as a symmetry line, (b) vector as a force, and (c) vector as an abstract tool.

Vector as a symmetry line. Selcan and Kiraz considered the vector as a symmetry line when performing geometric translations. Analyses of the initial semi-structured interviews revealed that both participants did not consider the direction and the magnitude of the vector when executing translations. For instance, when Kiraz was asked to perform a translation of a triangle based on a given vector, she used the vector as if it was a line of symmetry and performed a reflection (see Figure 2). Kiraz extended the vector and measured the perpendicular distances between pre-image points and the vector in order to specify the location of the image triangle.

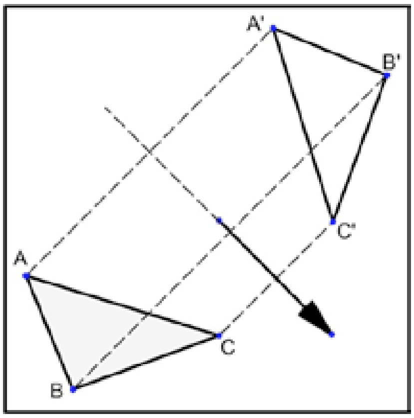


Figure 2. Selcan's execution of a translation based on a given vector

Vector as a force. Reyhan conceived the role of vector as a force in geometric translations. For instance, for the task which required her to perform a translation of a triangle based on a given vector, Reyhan placed the vector on point C to drag the triangle (see Figure 3). Reyhan stated that the dragging process would actually rotate the figure a little bit clockwise around point B, but this would still be a translation since the figure was moved to another place.

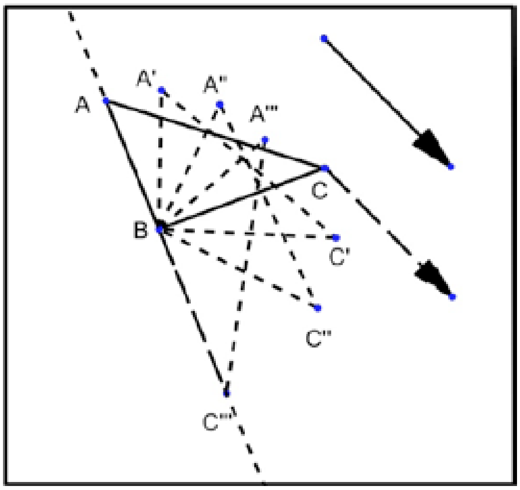


Figure 3. Reyhan's execution of a translation based on a given vector

Vector as an abstract tool. Samet conceived the vector as an abstract tool and was not sure about its role in geometric translations. Although he knew that vector had a direction and a magnitude, this knowledge did not lead him to conclude that a vector defines a translation.

Growths in Understanding of Translations in a Dynamic Geometry Environment

The analyses of the clinical interviews and the teaching episodes indicated that participants' understanding of translations evolved as they worked with GeoGebra. The findings of the study revealed that participants moved from motion understandings of translations towards an understanding of transformations as mapping of the plane onto itself. Although all the participants could not quite reach the mapping conception of translations, the findings of this study revealed factors that played a crucial role in participants' development of understanding, including the understanding of (1) translation vector, (2) relationships and properties of translations, (3) the domain, and (4) relationships between the plane and geometric figures. The findings further revealed that GeoGebra provided various opportunities for the participants to explore geometric translations, such as working with dynamic images, providing visual feedback, observing patterns, making conjectures and developing connections.

Conceiving translation as defined translational motion of a single geometric figure

Initial semi-structured clinical interviews revealed that all participants conceived translations as undefined motion with various levels of understanding. All participants conceived the domain as a single geometric shape and thought that translations would apply to only one geometric shape rather than all points in the plane. Futhermore, while Selcan and Kiraz considered the vector as a line of symmetry, Reyhan considered it as a force, and Samet had no idea about the role of vector in geometric translations (see Table 3).

The findings of the initial interviews indicated that the major reason for participants to conceive translations as undefined motion might be related to their incomplete understanding of the translation vector. Based on the analyses of the initial interviews, the researcher initially inferred that it would be plausible for participants to modify their thinking of translation as undefined motion through focusing on the vector. In order to achieve this goal, the researcher designed an instructional unit that included a set of tasks for each participant to explore the role of vector in translations using GeoGebra.

Developing meaning for the vector through the use of GeoGebra. During the first two teaching experiments, several tasks were provided to each participant focusing on performing translations. Initially, all participants were asked to predict the location of the image figures based on given vectors. Later, they were asked to use GeoGebra to test their assumptions and reflect on their predictions. The analyses of the first set of individual teaching experiments indicated that several features of GeoGebra influenced the participants' understanding of translation vector.

Specifically, participants were able to perform translations using GeoGebra and see the effect of the translation vector concurrently. This provided the first visual feedback for participants regarding the role of vector in translations and provided opportunity to reflect on their early predictions. For instance, for a translation task, Reyhan had initially conceived the vector as an external force and used it to rotate the pre-image (see Figure 4). When she actually executed the translation, she recognized that translation did not alter the direction of the figure and conjectured that translation should be a translational motion rather than a rotational motion.

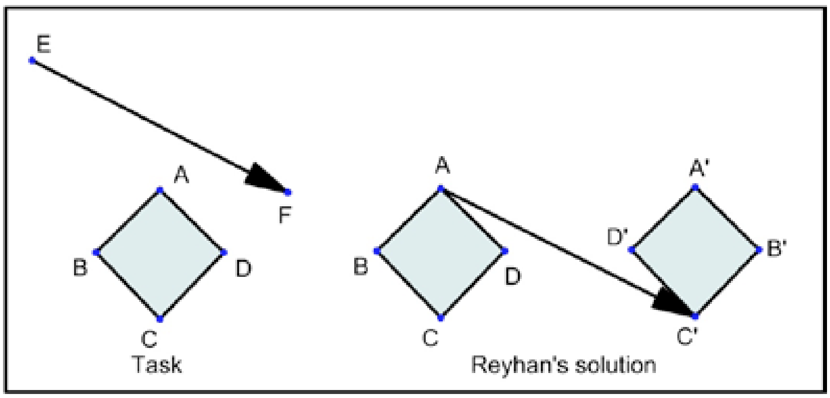


Figure 4. Reyhan's own way of performing a translation

Another participant, Kiraz, who conceived the vector as a line of symmetry, recognized that translation was a translational motion in the direction of the vector and would not change the orientation, shape, and the size of the pre-image by just looking at the result of translation when she executed the translation using GeoGebra.

However, not all the participants reached the proper conclusions through observing the result of a translation. For instance, after observing a set of translation tasks, Samet, who initially conceived a translation as translational motion and the vector as an abstract tool, concluded that translation might also be achieved through rotational motion and vector could be considered a pushing force. Therefore, additional experiments were needed beyond just making mere observations for the teacher candidates to explore the basic tenets of translations and the role of vector.

The drag mode of the program provided additional opportunities for participants to explore the characteristics of the translation and the vector. Teacher candidates used dragging for several purposes, such as to (a) observe the effect of dragging on the result of a translation, (b) test an assumption, and (c) verify a conjecture. Particularly, dragging the head or the tail of the vector helped participants recognized how the changes on the direction and the magnitude of the vector influenced the size, shape, and the location of the translation. For instance, although Samet initially thought that translation could be achieved through rotational motion, through dragging the head of the vector back and forward, he recognized that translation would not include rotational motion. Furthermore, Samet also verified his conjecture that changing the direction or magnitude of the vector would alter the location of image through dragging the head or the tail of the vector. Reyhan tested her conjecture that the length of the vector and the distances between the pre-image and image points would be the same through dragging the whole vector and placing it between pre-image and image points.

Measurement capabilities of GeoGebra also were helpful for teacher candidates to analyze properties of geometric translations. In general, the ways in which the participants used measures included testing and verifying conjectures. For instance, after performing a translation using a given vector, Kiraz made an assumption that the distances between pre-image and image points would be equal to the length of the vector (e.g., EE′=GH) and the distances between the end points of the vector and the pre-image and image points would be the same (e.g., EL=E′K) (see Figure 5). In order to test and verify her conjecture, Kiraz constructed a line that crossed the end points of the vector and then drew perpendicular lines to that line from the points E and E′. She then measured the distances and verified that her conjecture was correct (see Figure 5).

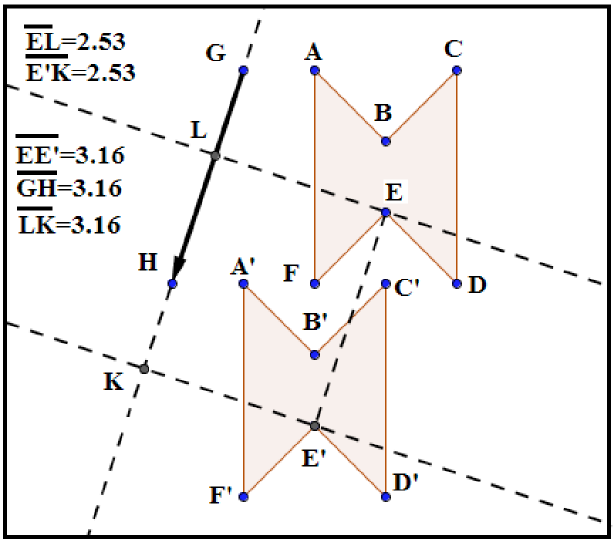


Figure 5. Kiraz’s verification of a conjecture

At the end of the first teaching experiment all participants considered the translation as motion of a single geometric figure on the plane defined by the direction and the magnitude of the vector. The findings of the first teaching experiment also revealed that the dynamic aspect of the program actually supported the teacher candidates' motion conception of translations. The researcher inferred that it would be plausible for teacher candidates to focus on the result of translation rather than the action of translation in order to progress from motion towards mapping understanding of translations.

Focusing on the effect of multiple translations. The second teaching experiment focused on the *effect* of multiple translations. By focusing on the *effect* idea, the researcher aimed at developing meaningful understanding for translation vectors and directing participants' attention more on the result of translation rather than the translation motion. In order to achieve this goal, participants were provided multiple vectors and asked to perform translations using the given vectors. For instance, for a given task (see Figure 6), when Reyhan was asked to reason about whether she could produce the same effect without performing multiple translations, she stated that instead of performing five distinct translations using the given vectors, she could achieve the same result using a single translation vector, which would be the resultant vector (see Figure 6). She also added that the order of translation vectors was not important since she could reach the same end-result even she would follow a different path.

The rest of the participants also provided similar responses. Participants indicated that the *effect* idea directed their attention on the result of a translation. Furthermore, participants indicated that as long as they knew the "location" of the image figure, they could identify an infinite number of translation vectors to achieve the translation. While they knew that translation vector indicates a path one needs to follow, they also considered that one could follow different paths and still reach the same location.

At the end of the second teaching experiment the participants continued to conceive translations as defined motion of single geometric figures. They began to conceive the translation vector as a *journey* (Watson, Spyrou, & Tall, 2002), which referred to the idea of moving a point or a figure from one point to another. However, the findings of the second teaching experiment also indicated that participants still conceived the points of the plane as independent physical objects, which could be manipulated on the plane rather than as locations in the plane.

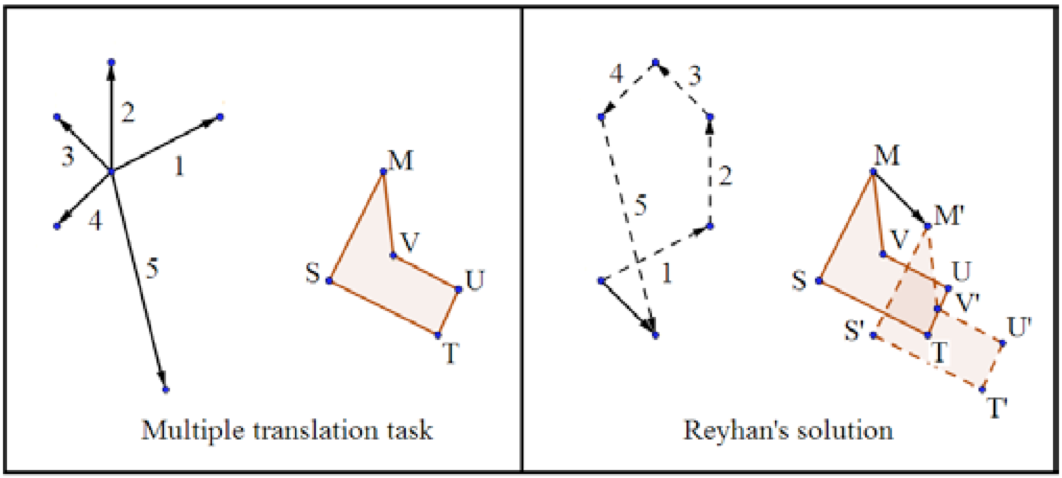


Figure 6. Reyhan's solution for a multiple translation task

Conceiving translation as defined motion of all points on the plane

After the analyses of the second teaching experiment, the researcher inferred that it would be plausible for teacher candidates to modify their thinking regarding the idea of domain for geometric translations and the relationships between geometric figures and the plane for the next teaching experiment.

Conceiving the domain for translations. The concept of domain as all points in the plane was new to all participants of the study. This became apparent when participants were asked to reason about multiple figures that were given for a translation. Participants seemed unsure about which figure to translate. In general, all participants considered selecting only one figure to translate rather than choosing all of them.

During the third teaching experiment, the researcher shared the knowledge regarding a translation applies all points in the plane with the participants. However, participants seemed unsure about how to use GeoGebra to apply translations to all points in the plane when executing translations. When participants were provided multiple geometric figures and asked to perform a translation using a given vector, initially they all selected only one figure to translate. One major difficulty for the participants applying this new knowledge seemed to be related to understanding the relationship between the plane and the geometric figures.

Understanding the relationship between the plane and the geometric figures. All participants conceived the plane and the geometric figures consisted of infinite number of points. However, when performing translations, the participants considered the geometric figures as independent from the plane rather than as part of it. Participants thought that plane could be considered as a background where geometric figures could be manipulated on it. Kiraz said: "When I translate a triangle, I am taking the whole figure and relocating it on somewhere else on the plane based on the direction and the magnitude of the vector".

In order to help participants visualize how one translates all points in the plane, during the teaching experiment, participants were asked to reason about the difference between moving a single object on a paper and moving the whole paper (Flanagan, 2001). All participants recognized that by moving the whole paper, everything in the plane moves concurrently. When participants were asked to think about how they could apply a translation to all points on the computer screen, they stated that they needed to select everything on the computer screen. Participants also recognized that when they applied a translation to all points of the plane, it would preserve the relative distances and angles among all points in the plane. When participants began to consider the geometric figures as part of the plane and translations apply all points in the plane, they also began to consider translation as the motion of the plane. Regarding the vector, although all participants began to conceive it as a parameter that would define a translation, they were reluctant to consider the vector as part of the plane. According to the participants, translation vector was just a tool for them to determine the direction and the magnitude of the translation.

Understanding translations as mappings. At the beginning of the fourth teaching experiment all participants still held the motion conception of translations even they considered the domain as all points in the plane and the vector would define a translation. The researcher conjectured that focusing on the points as locations on the plane rather than as physical entities would assist teacher candidates conceive translations as mappings rather than motions of the plane. Therefore, during the fourth teaching experiment, participants were directed to focus on the effects of translations on the locations of the pre-images through the use of coordinate system.

One difficulty participants encountered was how to conceive the coordinate system during translations. Participants seemed confused whether or not they should consider the coordinate system as part of the plane while performing translations. When they considered the coordinate system as part of the plane, they struggled how to describe the location of the image figures since they thought that the location of the coordinate system would also be influenced by the translation. After executing a translation, one of the participants, Kiraz stated: "The coordinate system should be independent from the plane otherwise we cannot determine the location of the image points." Through examining the locations of pre-image and image points, Kiraz also recognized that one determines new points as a result of translation. She said: "It is like defining new points based on the direction and the magnitude of the translation. You don't take the pre-image and relocate it somewhere else. You specify new points based on the vector and construct the image figure there. You do this for all points in the plane." Using the coordinate system and thinking about the points as locations helped participants focus on the effect of translation and began to conceive translations as mappings.

Discussion

The purpose of this study was to explore the nature of growth of prospective middle school mathematics teachers' understanding of geometric translations in a technology mediated environment using GeoGebra as a pedagogical medium. The findings of the study indicated that the availability of the dynamic geometry software supported teacher candidates' understanding of geometric translations. Specifically, the dragging and the measurement features of the program enabled teacher candidates to explore the properties of geometric translations, make conjectures, employ various strategies, and construct new understandings. Toward the end of the study, participants became more confident and were able to recognize, describe, execute, and represent translations.

The role of GeoGebra on learning the properties of geometric translations

The findings of the study showed that in the beginning of the study all the participants had an incomplete understanding of geometric translations and the translation vector (see Table 3). Several features of GeoGebra seemed to assist the participants in developing a more coherent understanding of geometric translations.

Providing immediate visual feedback. Immediate visual information that GeoGebra provided just after the execution of a translation seemed to assist some of the participants in making meaningful inferences regarding the properties of translations. For instance, Reyhan and Kiraz revised their understanding of vector just after they executed a translation using GeoGebra. Without manipulating the head or the tail of the vector, both participants were able to reason about the role vectors play in relation to translations. Furthermore, both participants recognized that a translation would preserve the direction, orientation, shape, and the size of the pre-image through observing the result of translation using GeoGebra.

However, the findings of the study also indicated that the mere observations of translation results were not sufficient for all participants to reach proper conclusions. While the participants tried to describe the actions they observe while using GeoGebra, they also generated and tested conjectures in order to gain further insights.

Constructing conjectures. GeoGebra provided the participants opportunities to explore some important questions (e.g., How would you explain the result of translation? and What can you do to test your explanation?) while encouraging participants to generate and test conjectures. For instance, after using the translation feature of GeoGebra to translate a figure, Samet initially could not decide whether or not translation could also be achieved through rotational motion. He conjectured that dragging the head of the vector would provide him some clues about how a translation could be achieved. He then manipulated the head of the vector and observed how the image figure behaved as a result of manipulation of the vector. Based on his observation Samet concluded that translation would only be achieved through translational motion. Selcan hypothesized that the distances between the pre-image and image points would be equal to the length of the vector. In order to verify her conjecture, she dragged the whole vector and placed it between the pre-image and image figure to show that her assertion was correct.

The role of GeoGebra on employing various strategies

The findings of this study indicated that teacher candidates employed two types of strategies in their activities with GeoGebra: reactive and proactive as coined by Hollebrands (2007).

> "Students who use the tool in a reactive manner might not know what to expect prior to acting; students who are using the tool proactively might have certain expectations of what they want to do with the technology, determine what actions will achieve their desired result, and then perform the action and reflect on the results that appear on the screen" (Hollebrands, 2007, p. 184).

According to Hollebrands (2007), learners' use of strategies appeared to be impacted "by their understandings of geometric properties and relations and their perceived affordances of the tool" (p. 188). In the initial teaching experiments, all participants employed reactive strategies to explore geometric translations. Dragging the head or the tail of the vector randomly to figure out the role of vector in translation was an example of a reactive strategy. Participants who used this strategy initially did not anticipate results from their action. The immediate visual feedback they received from the computer enhanced their understandings of geometric translations and their use of strategy was changed from reactive to proactive once they begin to construct new understandings regarding the relationships and properties of geometric translations. For instance, when Kiraz was trying to understand the role of vector in translations, she dragged the head or the tail of the vector without knowing what to expect as a result of her action. Once she began to see how the changes on the direction and the magnitude of the vector influenced the result of translation, her conceptions of translation and translation vector were rebuilt. Later Kiraz began to employ proactive strategies. For example, when Kiraz was asked to reason about a translation using multiple translation vectors, she stated that a resulting vector would produce the same effect. In order to make her thinking explicit, she used *head to tail* method to find out the resulting vector and then performed the translation using that single vector. Kiraz appeared to have a plan in advance when using GeoGebra based on the role of vector in translation. This strategy was different than her initial strategies and her construction of new understanding seemed to play a critical role in her choice of strategy.

Conclusion

The findings of the study indicated that through the help of GeoGebra teacher candidates progressed from motion-based reasoning towards a mapping conception of geometric translations. Initially, the dynamic feature of GeoGebra seemed to support the participants' motion conceptions. When participants observed the behavior of points under dragging, they conceptualized it as motions rather than as mappings of points. After the initial experiences with GeoGebra, all participants defined geometric points as motions of a single geometric figure based on the direction and the magnitude of the vector.

In some cases, the researcher purposely directed participants' attention to effect of translation and aimed to help participants focus on the end-result of the translation rather than the action of translation. While focusing on the result of translation seemed to help participants to focus on the end-location of image figures, participants continued to conceive translation as motions rather than as mappings. Focusing on the coordinate system and points as locations seemed to alter participants' conception from motion to mapping. Through GeoGebra's immediate visual feedback, participants were able to focus on the result of translation and see points as locations rather than as physical entities on the plane.

The findings of the study indicated that prospective middle school mathematics teachers predominantly hold a motion conception of geometric translations. Past research (e.g., Edwards, 2003; Glass, 2001; Harper, 2003; Hollebrands, 2003; 2007; Jung, 2002; Yanik, 2009) also indicated that pupils of different backgrounds and ages conceptualized transformations as mainly motion. Prior research further showed that some "conceptions are deeply embedded in students' minds" and can cause resistance for future learning (Aguirre, 1988, p. 216). The findings of the current study showed that transition from motion to mapping conception of translations was a challenging task for the teacher candidates. While GeoGebra supports the learning of relationships and properties of geometric translations, transition from motion to mapping conception takes time and requires carefully designed experiences. Although this study provided some ideas regarding teacher candidates' developing ideas of geometric translations, it is still unclear whether or not other teacher candidates would follow a similar path as shown in this study. Further studies are needed to explore what other components of technology-mediated environments support or inhibit the learning of geometric transformations.

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